THE OPTICAL THEOREM, A PURITY OF THE INITIAL STATES AND THE RISING TOTAL CROSS-SECTIONS*)

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A possibility of the violation of the optical theorem at high energies due to the mixed character of the initial states is discussed. A close connection of this idea with the MIT — bag model of hadrons is shown. The experimental evidence for the validity of the optical theorem is doubted. A plan of the investigations which could lead to the discovery of the optical theorem violation for strong interactions is presented.

1. INTRODUCTION

It is not a discovery to say that we have not yet a satisfactory theory of strongly interacting particles.

Instead, we have at our disposal a huge number of data, namely, the values of the various cross-sections for the exclusive and inclusive reactions. Consequently, we have a set of probabilities P_{if}, where P_{if} is the probability of observing a final state "f", if the initial particles are in a state "i". The state "i" of the two colliding initial particles (one from the beam, one from the target) is specified by a particular preparation of the beam and of the target.

What we need is a mathematical model of these scattering phenomena, in which to the initial and to the final states would correspond well defined mathematical quantities and to the scattering process mathematical operations, giving an unambiguous algorithm for the calculation of the different probabilities P_{if}.

A general belief is that in spite of the fact that we do not know this algorithm, a general framework of the future theory is known. It is a framework of the relativistic quantum field theory, or of the relativistic quantum mechanics.

For a general description of the scattering phenomena they essentially do not differ so much, namely, the initial "i" and the final "f" states are represented by the unit vectors $|i\rangle$ and $|f\rangle$ respectively, in the Fock type Hilbert Space H and the probabilities P_{if} are given in a form

$$P_{if} = |\langle f| S |i\rangle|^2 \tag{1}$$

where S is an unitary operator to be determined by a theory[1]).

In spite of this general belief everybody is aware of the possibility that at the distances below 10^{-13} cm the laws of the relativistic quantum mechanics can turn out to be inapplicable.

This is the reason why the models, based not on the quantum theory, but rather analogous to the classical theory of the continuous media have been constructed.

We have in mind various statistical or thermodynamical models, for example those of FERMI [1], HAGEDORN [2], FRAUTSCHI [3], JACOB and SLANSKY [4], and also Landau hydrodynamical model [5] which encountered many successes in fitting the data (look for a review and further references CARRUTHERS [6] and FEINBERG [7]).

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Recently a new promising model which combines both quantum and hydrodynamical intuitions has been constructed by CHODOS, JAFFE, JOHNSON, THORN and WEISSKOPF [8], namely so called MIT-bag model.

In a series of papers [9—14] we discussed a validity of the optical theorem and of some other more specific assumptions used in the theory or in the analysis of the experimental data. In this paper we are going to continue and broaden this discussion. For this reason we want to make here in the introduction few general comments about the experimental testing of physical assumptions.

Very rarely a single physical assumption can be directly tested experimentally. Usually one can compare with the experimental data only a whole set of assumptions (called a model) which leads to explicit analytic expressions P^t_{if} for P_{if}. Experimental values of P_{if} are known within the error bars and can be usually found consistent with different analytic formulae P^t_{if}. Very often models based on completely different physical assumptions lead to the P^t_{if} consistent with the data.

Therefore the agreement of the data with the predictions of a particular model, containing usually some free parameters not of the fundamental character, can not be treated as an ultimate proof of the correctness of this model.

In a different model, different questions are meaningful. A question: "What is the value of the slope of the Pomeron trajectory?" has a meaning in the reggeon calculus. A question: "How many partons seem to be in a hadron?" has a meaning in a parton model. Confronting a model with the data one gets meaningful answers to these questions. However, one should not forget (what happens quite often) that in future we may discover a model or a theory of strong interactions such that the above stated questions will have no meaning at all.

Let us illustrate it on an example. In a theory of ether one could pose meaningful questions about its properties and get meaningful answers from the experiment.

Many ether-theoretists were sure that the properties of the ether which they have discovered would be necessarily present in any future theory.

Now we know, that they were wrong so we should be more open minded and do not reject new models and hypotheses simply because they do not answer the questions which in our present models are meaningful.

Keeping this in mind we are going to discuss the following questions:

1) Can the optical theorem be violated?
2) Are the initial states in the scattering pure, if one neglects the impurities with respect to spin?
3) Can the optical theorem be tested?
4) How reliable are the extrapolations to the forward direction?
5) Do the total cross-sections rise?
6) How to test the purity of the initial states?

At the end of this lengthy introduction we would like to say that it is very probable that conventional assumptions made both in the theory and in the experimental analysis are correct and a criticism and hypotheses presented below will turn out to disagree with the experiment. However testing and eventually rejecting these hypotheses we can get a confirmation that we are searching for the theory of strong interactions in the correct direction.

2. THE OPTICAL THEOREM

In the non-relativistic quantum mechanics scattering of the particle beams is essentially represented as the scattering of the probability waves on the external potential. The evaluation of the various cross-sections is unambiguous due to the explicit use of the position representation $\langle x | \psi \rangle$ of the abstract state vector $|\psi\rangle$ associated with a quantum system.

In the relativistic quantum theory the position representation of $|\psi\rangle$ has no clear physical meaning for many reasons both theoretical and experimental. A long search for the reasonable relativistic position operator gave many important results (let us only mention NEWTON and WIGNER [15], WIGHTMAN [16], BARUT and MALIN [17] and FLEMING [18] contributions to the problem) but have not yet led to a satisfactory solution. A recent review of the investigations in this direction has been done by KALNAY [19]. This is why in the relativistic domain one is forced to use the quantum mechanics in a more abstract way.

A state ψ of one free particle of mass m, of helicity λ and momentum profile $\psi(p, \lambda)$ is represented by a vector $|\psi\rangle$ in the carrier space of the irreducible representation of proper Poincaré group

(2)
$$|\psi\rangle = \sum_{\lambda} \int \frac{d_3 p}{\sqrt{(p^2 + m^2)}} \psi(p, \lambda) \, |p\lambda\rangle ,$$

where $|p, \lambda\rangle$ is a helicity and momentum eigenstate (look for example WERLE [20]).

The same state interpreted as one-particle state of a free massive spinor field is represented by a vector

(3)
$$|\psi\rangle = \sum_{\lambda} \int \frac{d_3 p}{\sqrt{(p^2 + m^2)}} \psi(p, \lambda) \, a^+(p, \lambda) \, |\Omega\rangle ,$$

where $a^+(p, \lambda)$ is an appropriate creation operator acting on a vacuum $|\Omega\rangle$ [2]). By taking appropriate tensor products of the vectors (2) and (3) one constructs many particle states [20]. The probabilities P_{if} (1) calculated directly in both approaches factorize

(4)
$$P_{if} = A(\varphi_i) \, \tilde{P}_{if} ,$$

where $A(\varphi_i)$ depends only on the details of the initial profiles φ_i and $\tilde{P}_{if}$ depend only on the dynamics. To obtain a formula for the differential cross-section from the equality (3) one has to make additional assumptions which in view of the difficulties with the lack of the position representation may be unjustified. Namely, one relates the factors $A(\varphi_i)$ in (4) to the initial flux interpreting the Fourier transforms of the profiles φ_i as position probability amplitudes (look for explicit calculation MARTIN and SPEARMAN [21]).

According to the conventional interpretation

1) all pairs of the particles one from the beam one from the target (treated as point-like objects) have the same probability to meet in a space time point and then the same probability to produce a particular final channel of the reaction,

[2]) Also in the framework of the quantum field theory a question about the position of the field state has not a clear physical meaning.

2) the S operator is written in a form

$$S = I + iT, \tag{5}$$

where T has the interpretation of the scattering operator.

From the formula (5) and from the *interpretation* of T one gets that the probability amplitude for a free motion is always equal to 1 and the amplitude for the forward elastic scattering is always equal to $i\langle i|T|\, i\rangle$. The interference of these two amplitudes and the unitarity of S matrix imply the optical theorem

$$\operatorname{Im} f_{ii} = \sqrt{[\lambda(S, m_1^2, m_2^2)]}\, \sigma_{\text{tot}}, \tag{6}$$

where f_{ii} is in an obvious way related to the amplitude T_{ii}, λ and σ_{tot} denote a standard kinematical factor and the total cross-section respectively.

There is something mystic for us in this conventional interpretation, according to which the particles meeting in the same space-time point have a probability amplitude for a free motion *equal always to 1.*

Besides we do not like a theorem (6) for the following reasons:

1) it is a basic theorem for the practically unmeasurable quantity,
2) it excludes a possibility of the existence of strong interactions which do not have the elastic channel or at least those which do not have the forward elastic scattering. For these reasons *we doubt in the assumptions used in the proof.*

In fact, the assumption that the initial ensemble of two-particle states is pure has not been carefully tested. If one imagines hadrons to be the extended objects of the sizes of the order 10^{-13} cm, then, on the intuitive grounds, it seems very probable that in the scattering events we deal with mixed initial states. The hadron pairs may differ by a spatial configuration (for example by the value of the impact parameter) which can be crucial for the way they interact.

A common argument is that such a picture is in contradiction with the quantum mechanics. In our opinion it is only in contradiction with some interpretations[3]) of the quantum mechanics and with a simple extrapolation of the picture of the scattering of the non-relativistic probability waves on the external potential to the domain of the high energy physics. The problem of the scattering of the true extended objects has not been discussed in the framework of the quantum mechanics.

If one is attached to one's intuitions one believes that a new model of the scattering of the extended objects should have the following features. The initial states in the scattering should be a statistical mixtures with respect to some parameters α which cannot be controlled in the preparation stage of the experiment. The details of the interactions for which one has to construct a specific dynamical model, which may be very unconventional one, depend on the values of the parameters α for a particular pair of the colliding particles.

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[3]) Look the discussion of the various interpretations of the quantum mechanics made by BALLENTINE [22].

If α is interpreted to be an unmeasurable impact parameter b of the pair, then if $b > 2a$, where "a" denotes a radius of the hadron, then the particles do not interact strongly, if $b \leqq 2a$ the particles do interact strongly. In a case of strong interaction the outcome of any scattering event is not determined, thus the model of the strong interactions has to lead to the statistical predictions, namely, has to give the values of the probabilities $\tilde{P}_{if}$.

This, precisely, is a picture of the scattering phenomena in the hydrodynamical [5–7] and MIT-bag model [23]. In the first model highly inelastic head-on-collision has a hydrodynamical description, in the MIT-bag model [23] colliding bags, with confined colored quarks and colored Yang-Mills gluons inside, hit each other and make one bigger bag. Then the quarks from the different bags can exchange colored gluons, bag streches and the break up into elementary bags takes place.

Applying these dynamical models one has to remember that they cannot be used for the determination of the true total cross-sections $(\sigma_{\text{tot}})^{\text{true}}$ which, due to the principal assumptions, made about the sizes of the hadrons, have to be purely geometrical. In principle those sizes could depend on the laboratory energies, but we assume that is is not a case. Thus

$$(\sigma_{\text{tot}})^{\text{true}} = \sigma\,, \tag{7}$$

where σ is a constant value related to the assumed dimensions of the particles.

The measured (uncorrected) total cross-sections $\sigma_m(s)$ do depend on the energy "s" of the system in the center of mass reference frame, because

$$\sigma_m(s) = \sum_{f'} P_{if'}(s)\,\sigma\,, \tag{8}$$

where $P_{if'}(s)$ denotes a probability of finding an experimentally distinguishable state f', if the strong scattering had taken place and if the initial state was "i".

Let us note that always

$$\sigma_m(s) \leqq \sigma\,. \tag{9}$$

A dynamical model has to be arranged to give immediately the probabilities $P_{if'}$ not the differential cross-sections. This fact has not been recognized by FRANCIS LOW in his stimulating discussion of the scattering of the MIT-bags [23].

It seems to us that we have found a way to show how such a picture of the scattering of the extended objects can be made consistent with the general ideas of the quantum theory [24].

Two years ago we had an idea that due to the extended character of the particles the initial states in the scattering should be mixed. We stated it [9, 11] as a general hypothesis, to be tested by the experiment. Such a general hypothesis can be, in principle, tested experimentally [12, 14] even, if the type of mixing is unknown.

We have succeeded to embed this hypothesis in the general "kinematical" framework of the relativistic quantum mechanics. We have shown [9, 11] that one can construct S-matrix description of the scattering without proving the optical theorem.

The idea of our approach is the following:

I. We assume that the Hilbert space H of the vector states is a direct sum of three subspaces

$$H = H_1 \oplus H_2 \oplus H_3 \,. \tag{10}$$

The subspace H_1 contains all initial two-particle states $|\alpha, \beta\rangle$ which have the values of the parameters α in some set A. The subspace H_2 contains all the remaining two-particle states. The subspace H_3 contains all n-particle states which remain.

II. If the initial state is from the H_1 it interacts strongly, if it is from the H_2 it evolves freely.

III. The realizable initial states have to be represented by the density matrices

$$\hat{D}_i = \sum_{\alpha} \varrho(\alpha) \, |\alpha, \beta\rangle \langle\alpha, \beta| \,, \tag{11}$$

where $\varrho(\alpha)$ denotes a probability of the appearance of a state $|\alpha, \beta\rangle$ in the initial mixture, β denotes other quantum numbers.

IV. The S operator has a form

$$S = I \oplus S^{\sim} \,, \tag{12}$$

where $S^{\sim} = S|_{H_1 \oplus H_3}$ is a *unitary scattering operator.*

Later everything is standard. The probabilities P_{if} (1) are given by a formula

$$P_{if} = \text{Tr}\,(|f\rangle \langle f| \, \hat{D}_f) \,, \tag{13}$$

where $|f\rangle$ is a state vector of the final state and $\hat{D}_f = S\hat{D}_i S^+$.

In the first paper [9] we took for the parameters α some additional parameter ξ for the two-particle states.

In the second paper [11] we did not introduce any new quantum number and we assumed that $\alpha = \{J, \Lambda, l, \sigma\}$ where J denotes the total invariant spin, Λ — its projection on the direction of the total linear momentum, l — the relative orbital momentum and σ — the resultant spin of the particles. The interacting initial states in this model were defined by an inequality $l \leqq pa$ where "a" — is the effective range of the strong interactions and ϱ is the average relative linear momentum.

After this explicit construction the claims, that one can not have the probability conserved, the unitary S matrix and the optical theorem violated, turned out to be unjustified.

Thus, the optical theorem stopped to be an absolute truth and should be carefully tested. The need of the careful tests of the optical theorem was also indicated in other context by BELL [25] and EBERHARD [26].

3. EXPERIMENTAL SITUATION

As we have already stated in the section 2, the optical theorem can not be directly tested experimentally. It is our deep conviction that any "direct" tests via the extrapolation of the observed elastic differential cross-sections to the forward direction cannot *decide*, whether the optical theorem is correct or not. All "direct" tests can only show a consistency of the optical theorem with a set of other assumptions, which necessarily have to be used in the extrapolation procedure.

An important theoretical and experimental question, if one wants to extrapolate, is a question "how to do it?" If we had a successful theory, for the observed reaction region, then the extrapolation to the forward direction could be made with its help. Since we do not have such a theory, the extrapolation procedures used by different authors depend on their personal taste. However, one should remember that, if the parametric formula used to fit the $d\sigma/dt$ is not correct, then the fit can be meaningless in spite of the reasonable χ^2. A careful discussion of these problems has been recently done by FRIEDMANN [25].

The extrapolation to the forward direction has been always made with a help of some "reasonable" formula, containing some free parameters. The parameters have been fitted under the constraint that the optical theorem holds. One can ask a meaningful question how the unconstrained fit agrees with the optical theorem.

To answer this question one can test the inequality which follows from the optical theorem

$$\left(\frac{d\sigma}{dt}\right)^{\varrho}_{t=0} \geqq \frac{(\sigma^{\varrho}_{tot})^2}{16\,\pi\hbar} \tag{14}$$

where $(d\sigma/dt)^{\varrho}_{t=0}$ and σ^{ϱ}_{tot} denote the differential elastic cross-section and the total cross-section respectively for the initial particles in the spin state described by the density matrix ϱ.

A careful examination of this question was done by EBERHARD [26] who wanted to test Bell's suggestion [27] and his own model of the non-unitary S operator [28].

A discrepancy of about 5 to 10% have been found for π^-p and π^+p between 0·5 and 2·6 GeV/c and between 8 and 26 GeV/c [26], no discrepancy was found for π^-p at 1·0, 1·5 and 2.0 GeV/c by EBERHARD et al. [29].

Searching for the violations of the inequality (14), we found in the ENSTRÖM et al. [30] $N\bar{N}$ compilation their best unconstrained fits to the different author's data for $d\sigma/dt$. The exponential formula $A \exp(-B|t|)$ was used. The fitted values for A, for $1 \leqq p \leqq 2$ GeV/c, vary between 78 and 1018. We could easily find the values of A violating dramatically the inequality (14). We have pointed out many reasons which could cause such an effect. It was justly pointed out by HOHLER and KROLL [31] that most of the "violations" can be attributed to the fact that the data start at too large values of t and the fits to the forward direction seem to be quite meaningless. The only "violating" data which survived are the data of PARKER [32]. We display by the agreement of the dispersion relations for pp with the data, since in the experimental analysis many assumptions are made to get such a consistency [13]. The experimental and theoretical status of the dispersion relations for pp and K^+p is not very satisfactory, what was recently explicitely pointed out by DUMBRAJS and STASZEL [34].

We agree with HÖHLER and KROLL that there is no experimental evidence for the optical theorem violation, but on their question "Are there reasons to doubt the optical theorem?" – we reply – "Yes, there are many".

The main idea of the paper [10] was not to prove the violation, but to encourage people to look for and to publish the data which "disagree" with the optical theorem.

The question arises how one should struggle with the optical theorem (if one is such a bad person that one wants to).

The plan of the battle is:

1) To show that the violation of the optical theorem can be consistent with the data.

2) To show that the interpretation of T as a scattering operator which leads to an assymetry between the upper limits for the partial inelastic and elastic differential cross-sections is not confirmed experimentally, namely, that one can always fit the elastic data with

(15) $$0 \leqq \sigma_J^{el} \leqq \frac{\pi}{p^2}(2J + 1) .$$

3) To show that the initial states are not pure in the sense discussed in section 2.

4) To develop dynamical models of "really" extended hadrons. Now we want to present the idea of the purity tests [12, 14].

4. PURITY TESTS

If the initial ensemble of two-particle states is pure then every pair of colliding particles have the same probability to choose a particular final channel of the reaction. From the formula (3) one can see that the relative probabilities $P_{ff'}$

(16) $$P_{ff'} = P_{if}/P_{if'} = P_{\tilde{i}f}/P_{\tilde{i}f'}$$

do not depend on the details of the initial momentum profiles. The best estimates to the $P_{ff'}$ are the ratios $I_{ff'}$ of the appropriate counting rates I_f and $I_{f'}$

(17) $$I_{ff'} = I_f/I_{f'} .$$

The probability density function for the distribution of the measured values $I_{ff'}$ should not depend on the macroscopic details of the momentum profiles (change of the macroscopic details does not spoil the factorization in the formula (3)). According to the conventional interpretation the changes of the profile functions correspond to the change of the geometry of the experiments and or to the change of intensity of the initial beams. Therefore in two nearly identical experiments (or in two runs of the same experiment) the values $I_{ff'}$ should be distributed according to the same law. We call the experiments nearly identical, because we assume that they can only differ with respect to the fluctuations of the shapes of the initial profiles, what in the convential language, means the fluctuations of the initial flux. Such fluctuations should have no impact on the values $P_{ff'}$ fixed by the dynamics.

If the initial states are mixed in the sense of section 2, then in nearly identical experiments we can have different sets of the particle pairs differing by the values of the parameter α. In this case the probabilities P^1_{if} and P^2_{if} are expressed in the following way

$$P^1_{if} = \sum_\alpha \varrho^1(\alpha)\, P_{\alpha f}\,, \tag{18}$$

$$P^2_{if} = \sum_\alpha \varrho^2(\alpha)\, P_{\alpha f}\,, \tag{19}$$

where $\varrho^i(\alpha)$ denote the probability of finding in the initial state a particle pair indexed by α, and $P_{\alpha f}$ denote the probability of production of the final state "f", for the initial state labelled by α. We have to accept that the $\varrho^1(\alpha)$ can be different from the $\varrho^2(\alpha)$ since the values $\varrho(\alpha)$ do depend on the macroscopic details of the initial profiles. Thus the measured values $I_{ff'}$, can be expected to be differently distributed.

Therefore purity tests consit on testing a non-parametric hypothesis [12, 14] H_0,

$$H_0 : f_1(X) = f_2(X)\,, \tag{20}$$

where $I_{ff'}$ are treated as the observations of some random variable X, $f_1(X)$ and $f_2(X)$ denote the unknown probability density functions of X in the experiment 1 and 2 respectively.

Since we do not want to discover the mixed character with respect to the spin we should measure such $I_{ff'}$, that, on the theoretical grounds, the $P_{ff'}$ seem not to depend on the spin.

A particular proposition for $I_{ff'}$, has been made [12], an extensive review of the statistical methods which can be used to test the hypothesis (20) has been prepared [14]. A particular purity test for π^-d multiplicity distributions is now in preparation by Gajewski and myself [35].

5. RISING TOTAL CROSS-SECTIONS

At the end we want to comment shortly on possible unconventional interpretations of the observed rise of the total cross-sections [36–42]:

1) Due to the uncertainities of the extrapolations to the unobserved experimental region we do not trust the small errors quoted in [36–42], in fact there is a possibility

that the additional theoretical errors due to the choice of the inappropriate parametric formulas for the elastic $d\sigma/dt$ are present [13, 25, 43].

2) Only the rise of the inelastic total cross-section can be clearly seen in [36–42].

3) In view of the model of an extended hadron [1–8, 23] the observed rise of the σ_{tot} could be due to the real rise of the inelastic total cross-section at the expence of the elastic total cross-section (in the unobserved experimental region).

4) In view of the impurity idea [9, 11] in the ISR experiments we can deal with a slightly different mixtures in the initial states.

6. CONCLUSIONS

We have simple purity tests, we have a lot of data. Purity tests are simple, cheap and amusing, they can allow us to test a fascinating idea of the extended hadron [1–8, 23][4]). Let us make them!

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