Unitarity without the Optical Theorem.

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The general belief is that the unitarity of the S-matrix necessarily implies the optical theorem. The unitarity of the S-matrix is derived from the conservation of probability. The conservation of probability is an obvious and unquestionable truth, so the optical theorem is believed to have a similar status.

However, the derivation of the optical theorem is based also on some additional less obvious assumptions which can, but not necessarily need, be true for high-energy strong interactions. Thus a careful analysis of the theoretical and experimental status of the optical theorem in strong-interaction physics is needed. In our opinion this is important just now, when the optical theorem is treated as one of the main methods in ISR physics (1).

One can argue that the objective truth of the optical theorem is proved by many successes of its applications. In our opinion such a point of view is not completely justified, since we know that one can achieve agreement with the experimental data using many completely different phenomenological models. We claim that the optical theorem is not checked satisfactorily enough because it is believed to be the main ever-true constraint on the reaction amplitudes.

The need of tests on the optical theorem was, to our knowledge, first indicated by BELL (2) and EBERHARD (3,4) in a different context. In the derivation of the optical theorem, besides the conservation of the probability, enter also the following assumptions, on which we centre our discussion:

1) *the purity of the initial ensemble of two-particle states* (besides the allowed impurity with respect to spin variables),

2) *the interference of the amplitude for the free motion with the forward scattering amplitude* implied by the cluster decomposition of the S-matrix, $S = I + iT$, where T is interpreted as the scattering operator.

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(2) J. S. BELL: *Remark to Sussmans' paper at the International Colloquium on Issues on Contemporary Physics and Philosophy of Science* (Philadelphia, Pa., 1971).

(3) P. H. EBERHARD: CERN preprint 72-1.

(4) P. H. EBERHARD: *Nucl. Phys.*, **48** B, 333 (1972).

In our opinion *neither of these assumptions is obvious and both require experimental verification.* The first is equivalent to the statement that two-particle states are completely described by their linear momenta and appropriate *conventional* internal quantum numbers, the second descends from the picture of the scattering of probability waves on the short-range potential in the nonrelativistic quantum mechanics.

One cannot exclude the possibility that for the complete description of two particles in the scattering process an additional parameter ξ may be needed.

On the other hand, although in the experiment one cannot see the difference between the elastic forward scattering and the free motion, we feel that they correspond to completely different physical situations. In the first case a beam particle and a target particle interact strongly, in the second they do not « see » each other.

If we can divide our ensemble of scattering events into two « subensembles »: a strong-scattering and a free-motion subensemble (by free motion we mean lack of strong interactions; we cannot switch off the electromagnetic ones), then we have a mixed quantum ensemble.

Therefore, assumptions 1) and 2) are in some way interrelated and they may turn out to be incorrect at the same time. We visualize this on a simple (not necessarily realistic) model.

Let ξ have only two values: 1 and -1. If $\xi = -1$ for a two-particle state, they do not interact at all, if $\xi = 1$, they interact strongly. If our initial ensemble is a mixture of two such kinds of two-particle states, according to quantum mechanics we must use density matrices to describe initial and final states.

Let an initial density matrix $\hat{D}_i$ be

$$\hat{D}_i = c(+)|i, 1\rangle\langle i, 1| + c(-)|i, -1\rangle\langle i, -1| , \tag{1}$$

where $c(+) + c(-) = 1$.

Now the unitary $\hat{S}$-matrix acts in the Hilbert space of states $H = H_1 \oplus H_{-1} \oplus H_f$, where H_1 and H_{-1} are incoherent subspaces of H spanned by the vectors $|i, 1\rangle$ and $|i, -1\rangle$, respectively, and $|i, 1\rangle = |i\rangle \otimes |1\rangle$, where $|i\rangle$ denote the usual two-particle states (appropriately normalized). H_f is the many-particle Hilbert space. The operator $\hat{S}$ has the following structure: $\hat{S} = I \oplus \hat{S}^\sim$, where $\hat{S}|_{H_1 \oplus H_f} = \hat{S}^\sim$ and $\hat{S}^\sim$ is unitary on this subspace. The $\hat{D}_f$ final density matrix is equal to $\hat{D}_f = \hat{S}\hat{D}_i\hat{S}^\dagger$ and the probability P^+_{ii} of finding the final state $|i, 1\rangle$ is $P^+_{ii} = \mathrm{Tr}(\hat{D}_f|i, 1\rangle\langle i, 1|)$. The probability of finding the final state $|i, -1\rangle$ is $P^-_{ii} = \mathrm{Tr}(\hat{D}_f|i, -1\rangle\langle i, -1|)$. The probability of finding one of those states is an incoherent sum $P^+_{ii} + P^-_{ii}$. The probability P^+_{fi} of finding a nonforward state $|f\rangle$ is

$$P^+_{fi} = c(+)\left|\langle f|\hat{S}^\sim|i, 1\rangle\right|^2 . \tag{2}$$

If we were able to purify our initial ensemble, then we could use the normal description of the pure initial states $|i, 1\rangle$ and the unitary $\hat{S}^\sim$ operator. The matrix elements $\hat{S}^\sim_{fi}$ would have the interpretation of the true reaction amplitudes.

The problem is how to purify the initial ensemble. To do this, the experimental data must be analysed in the following way. One has to pick up at first only scattered cases, by definition they have $\xi = 1$. One cannot show which of the unscattered tracks correspond to $\xi = 1$ or $\xi = -1$. However, for all future needs, one need only know the number of hypothetical forward-scattering cases. To find this number, one can assume the continuity of the angular distribution of the elastic scattering for $\theta \to 0$ and find the hypothetical number x of the forward events taking the limit $N(\theta)/(N + x) \xrightarrow[\theta \to 0]{} x/(N + x)$, where N is the total number of the nonforward cases.

The reaction amplitudes $\hat{S}^{\sim}_{fi}$ are strictly connected with the probability densities $|\hat{S}^{\sim}_{fi}|^2 \sim N_f/(N+x)$, and all specific dynamical information should be contained in them. Using them, one can in principle calculate all the relative cross-sections σ_{ch}/σ_{tot}, inclusive cross-sections, polarization of the final states and so on. However, they cannot be used to calculate the total cross-sections. *Thus* $\hat{S}^{\sim}_{ii}$ *are not connected with* σ_{tot} *and one does not have the optical theorem. Of course one has the conservation of probability* in the form

$$\sum_{ch} \sigma_{ch}/\sigma_{tot} = \sum_f\!\!\!\!\!\!\int |\hat{S}^{\sim}_{fi}|^2 = 1 . \tag{3}$$

Let us note that we can introduce an operator $\hat{T}^{\sim}$ such that $\hat{S}^{\sim} = I + i\hat{T}^{\sim}$, but now the amplitudes $\hat{T}^{\sim}_{ii}$ do not have the physical meaning of the forward scattering amplitudes, and also $i\hat{T}^{\sim}_{fi} \neq \hat{S}^{\sim}_{fi}$ for all $|f\rangle$ which are not orthogonal to $|i\rangle$.

In our approach we can imagine such strong interactions that the forward elastic scattering disappears, $\hat{S}^{\sim}_{ii} = 0$, in agreement with all postulates. In the usual approach such a theoretical possibility was excluded due to the optical theorem.

Summarizing, in our model we introduce a new parameter ξ, a mixed initial ensemble and the scattering part of the $\hat{S}$-matrix, $\hat{S}^{\sim}$, which is the *unitary operator* because $\hat{S} = \hat{S}^{\sim} \oplus I$, in contrast with the usual formula $\hat{S} = iT + I$. Further, we are only interested in the description of the subensemble of events in which beam and target particles interact strongly.

One could ask what could be the classical counterpart of the parameter ξ. The parameter ξ can correspond to the classical parameter

$$\xi' = \operatorname{sgn}\left(\min_t \left(-|\bar{x}_1(t) - \bar{x}_2(t)| + 2R\right)\right)$$

where $\bar{x}_1(t)$ and $\bar{x}_2(t)$ are classical trajectories of the particles in the scattering process, t is the time, R is the range of the strong interactions $\sim 10^{-13}$ cm.

The impurity of the initial ensemble of the type considered above has many other consequences besides a possible violation of the optical theorem and can be experimentally tested. The possible direct tests of this impurity will be discussed in a forthcoming paper.

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