ON SOME NEW TESTS OF COMPLETENESS OF QUANTUM MECHANICS

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It is observed that in a theory with supplementary parameters (TSP) each pure quantum ensemble is mixed with respect to these parameters. New efficient purity tests of quantum ensembles are proposed.

About 50 years ago Einstein, Podolsky and Rosen (EPR) [1] posed a question about the completeness of quantum mechanics. EPR wanted to challenge the statement that QM gives the most complete description of an *individual* physical system. In the so-called statistical interpretation of QM [2], consistent with their point of view, a wavefunction describes not a state of an individual physical system but a state of an ensemble of identically prepared physical systems and the wavefunction reduction is a passage from the description of the whole ensemble of these systems to the description of a subensemble satisfying some additional conditions.

The statistical interpretation of QM leaves place for the hypothesis of the existence of supplementary parameters which determine the behaviour of a particular physical system. Many theories with supplementary parameters (TSP) have been proposed and studied [3].

Theoretical and experimental analysis of the Bell inequalities [4,5], in particular the results of the experiments realized by Aspect et al. [9,10], seemed to indicate that, if a TSP wants to explain the data, it has to violate Einstein's separability (for a review see refs. [6–8,10]).

However, in view of some new arguments [11–14] we judge that the Aspect experiments rule out a large class of TSP but do not allow one to conclude that quantum mechanics is complete. Not being the advocates of any particular TSP we want to indicate new tests, which may be useful to verify the completeness of quantum mechanics.

The main feature of any TSP is that the quantum pure ensembles become mixed statistical ensembles of the individual systems characterized by the different values of these new parameters. There is a principal difference between a pure statistical ensemble and a mixed one. The pure ensemble is homogeneous, a mixed one should reveal a fine structure. To see this point clearly we give here a reasoning leading to the operational definition of the pure state [#1] and of purity tests [16].

Let O be a stable source of particles and γ a measuring device of some physical observable γX. A set $S=\{x_i;\ i=1,\dots,m\}$, where x_i denote the measured values of γX for m particles produced by a source O, may be interpreted as a sample drawn from some unknown statistical population of the random variable X associated with the observable γX. The probability density function $f(x)$ of X and its cumulative distribution function

$$F(x)=\int_{-\infty}^{x} f(x')\,\mathrm{d}x'$$

are unknown, but mathematical statistics gives us the means to estimate their main characteristics from the sampling density function or from the empirical distribution function $F(m,x)=n(x_i\leqslant x)/m$, where $n(x_i\leqslant x)$ is the number of observations from S smaller than or equal to x.

Let b_i be a beam of m_i particles produced by

[#1] An extensive discussion of the purity of the beams may be found in ref. [15].

the source O in the time interval $[t_i, t_i + \Delta t]$ and S_i a sample obtained by measuring γX on the beam b_i. We may also obtain other families of the beams $b_i(j)$, where j denotes the jth beam intensity reduction procedure applied to the beam b_i. Measuring γX on the beams $b_i(j)$ we obtain the new samples $S_i(j)$. We state that the beams produced by the source O are pure and described by a pure quantum state, if we cannot reject the hypothesis H_0: all the samples S_i and $S_i(j)$ for different values of t_i and Δt are drawn from the same unknown statistical population of the random variable X.

There are many statistical non-parametric compatibility tests which may be used to verify the hypothesis H_0. They were extensively reviewed [17] and the examples of their applications were given in a different context [18]. The purity tests may be used to analyze any beam which should be pure according to quantum mechanics and which is suspected to be mixed, if the hypothesis of the supplementary parameters is considered (one can study for example whether in the Fabrikant- or Janossy-type experiments [19] the interference pattern is built up in a regular way).

If the purity of the quantum "pure" ensembles is confirmed, then the statement that quantum mechanics gives a complete description of the individual systems will be proven. The completeness should be understood in the sense that the only predictable and reproducible characteristic of a physical system is: being a member of a given pure ensemble having the properties predicted by quantum theory.

Talking about purity tests we do not propose a particular experimental arrangement because the non-parametric compatibility tests may be used to analyze data from any experiment in which the samples $S_i(j)$ may be found and compared. In this letter we want only to point out that this comparison may provide new direct tests of completeness of quantum mechanics if the experiment is carefully chosen.

To be more specific let us use the language familiar to experimental physicists. In each experiment we have several runs. The reported experimental results are the statistical averages of the data obtained in the different runs. If the analyzed beam of particles is mixed and the individual particles are characterized by some supplementary parameters not controlled in the experiment it *may* occur that the statistical distribution of these parameters differ from run to run. The incompatibility of the different runs is not a new phenomenon in experimental physics, it is usually called statistical fluctuation. The experimentalists look carefully for the reasons for such fluctuations to find an indication how to readjust the data. If the reason is not found and the fluctuation is significant the data of the "bad" run are eliminated. In simple words the purpose of the purity tests is a systematic search for such inexplicable fluctuations by more detailed comparison of the histograms (of some physical variable independent of the initial intensity of the beam) obtained in the different runs of the same experiment. For this comparison various non-parametric compatibility tests may be advocated such as the Wilcoxon–Mann–Whittney test, normal scores test, rank or run tests [17] (for the run tests time ordering of the events in the experiment has to be known).

To start with, one can perform the purity tests on the data from the experiments of Aspect et al. The other suitable data are those coming from the beautiful neutron interferometry experiments (for a review see ref. [20]). One may use in these experiments natural beam intensity reduction procedures: static or time-dependent absorption [21]. The new experiments are running and the others are programmed [22]. The analysis of their data may be supplemented by the tests proposed above.

To conclude, we hope that the purity tests will provide a more comprehensive answer to the 50 years old EPR-question, which, being philosophical, is in fact an experimental one. The most expected answer is: "No evidence for incompleteness".

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