PITOVSKY MODEL AND COMPLEMENTARITY

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A new formulation of the deterministic local Pitovsky model, able to reproduce the predictions of quantum mechanics for distant polarization correlations and to give some insight into Bohr's complementarity, is given.

In this letter we want to discuss in some detail and in a new way the Pitovsky model (P-model) [1,2]. We do not aim to advocate this particular subquantal description of the EPR phenomena but we find this model interesting because its existence provides a proof that the results of the experiments of Aspect et al. [3] do not lead to conclusive statements concerning the completeness of quantum mechanics (QM) [4]. We are convinced that the first formulation of the P-model was not well understood. Moreover, in our new formulation the model does not look like a mathematical pathology and gives, paradoxically enoug, some insight into the notion of Bohr's complementarity [5].

In the P-model any spin-$\frac{1}{2}$ particle is described by a spin function s defined on a set $S^{(2)}$ of all unit vectors in the three-dimensional euclidean space ($s(x)=\pm\frac{1}{2}$ for $x\in S^{(2)}$). With different particles one associates different spin functions from a set F_0 having some remarkable properties to be specified later. Let us introduce some important subsets of F_0:

$$F_+(y)=\{s'\in F_0\,|\,s'(y)=\tfrac{1}{2}\}$$

and

$$F_-(y)=\{s'\in F_0\,|\,s'(y)=-\tfrac{1}{2}\}$$

which may be defined for each $y\in S^{(2)}$. We define also the probabilities $p(x^+)$ (probability of finding $s(x)=+\frac{1}{2}$ if s is randomly chosen from the set F_0) and $p(y^+, x^+)$ (probability of findings $s'(x)=+\frac{1}{2}$ if s' is randomly chosen from the set $F_+(y)$).

The main result [‡1] of the P-model may be stated in the following way: there exist spin functions and the set F_0 such that for each x and $y\in S^{(2)}$, $s(-y)=-s(y)$, $p(y^+)=\frac{1}{2}$ and $p(y^+, x^+)=\cos^2(\theta_{yx}/2)=p(x^+, y^+)$, where θ_{yx} is the angle between the vectors y and x ($0<\theta_{yx}<\pi$). One finds also the probabilities $p(y^-)=\frac{1}{2}$ and $p(y^+, x^-)$ $\sin^2(\theta_{yx}/2)=p(x^-, y^+)$, where $p(y^-)=p((-y)^+)$ etc. A device, chracterized by a vector $y\in S^{(2)}$, which transmits a particle if $s(y)=\frac{1}{2}$ and absorbs it otherwise is called an ideal polarizer Y. After the act of transmission by the polarizer Y the particle's spin function s is changed into another function $s'\in F_+(y)$ (for example $s'(y)=s(\alpha(y))$, where α is a rotation around the direction y).

Let us imagine a beam b of particles characterized by spin functions s randomly chosen from the set F_0. If we analyze this beam using the polarizer Y we obtain a beam b_Y of reduced intensity (the passage probability being simply equal to $p(y^+)$). The beam b_Y is composed of particles described by spin functions s' randomly chosen from the set $F_+(y)$. If we have two ideal polarizers X and Y, characterized by the orientation vectors x and y respectively, we may estimate the transmission probability $p(Y, X)$ for these two polarizers (probability for the particle which passed by the polarizer Y to be transmitted by the polarizer X) by the measurement of the intensities of the beam b_Y before and after its passage by the polarizer X. It is not difficult to see that $p(Y,$

[‡1] This main result and the only one we need is theorem 1 proven on pp. 2324,2325 of ref. [2].

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X) $=p(y^+, x^+)$ and that $p(y^+, x^+)=\cos^2(\theta_{yx}/2)$. Let us consider the ensemble $F_+^\beta(y)$ of all spin functions s which are obtained from a given spin function s_β ($s_\beta(y)=+\frac{1}{2}$) by an arbitrary rotation α around the vector y. The probability that $s(x)=+\frac{1}{2}$ if s is drawn at random from $F_+^\beta(y)$ is equal to the probability that $s_\beta(x)=+\frac{1}{2}$ if x is drawn at random from the circle $c(y, \theta_{yx})$ ($c(y, \theta_{yx})=\{x \in S^2 | x \cdot y = \cos\theta_{yx}\}$). Since in the P-model the last probability is equal to $\cos^2(\theta_{yx}/2)$ for any value of β and since $F_+(y)=\cup_\beta F^\beta(y)$ we obtain $p(y^+, x^+)=\cos^2(\theta_{yx}/2)$.

To describe the idealized EPR-experiment one may represent each pair of particles 1 and 2 by the corresponding spin functions s_1 and s_2 respectively, with $s_2=-s_1$ and s_1 randomly chosen from the set F_0. Let us imagine that the particles 1 are analyzed by the ideal polarizer Y and the particles 2 by the ideal polarizer X. To estimate the probability $p_{12}(y, x)$ (probability that a particle 1 passes through the polarizer Y and a particle 2, correlated with the particle 1, passes through the polarizer X) a coincidence technique must be used, the passage of a particle 2 by the polarizer X is registered only if the corresponding particle 1 has passed through the polarizer Y. Therefor the particles 2 taken into account are those characterized by functions $s_2' \in F_-(y)$ and

$$p_{12}(y, x) = \lim_{N\to\infty} \frac{1}{N} \sum_{j=1}^{N} [s_j(-y)+\tfrac{1}{2}][s_j(x)+\tfrac{1}{2}]$$

$$= \lim_{N\to\infty} \frac{n}{N} \left(\frac{1}{n} \sum_{k=1}^{n} [s_k(x)+\tfrac{1}{2}] \right)$$

$$= p(y^+)p(y^-, x^+) = \tfrac{1}{2}\sin^2(\theta_{yx}/2) , \qquad (1)$$

where $s_j \in F_0$, $s_k \in F_-(y)$ and $n=n(N, y)$ is the number of s_k in an experimental run of N particles 2. This is exactly the prediction of quantum mechanics for these experiments. Thus this model violates the Bell inequalities [6] without violating the einsteinian separability.

Since we have the impression that the answer of Pitovsky [7] to the criticism [8] of the first version of the P-model has not been understood, we make the following points to clarify the issue:

(1) To deal with realistic experiments the model discussed so far must be extended. An ideal polarizer does not exist. With a realistic polarizer $Y_R = A$ we may not associate a unique orientation vector a, any other vector $y \in O_A$ ($O_A = \{y \in S^{(2)} \| 1 - y \cdot a| < \epsilon_A\}$, where ϵ_A is a small real positive number and $y \cdot a$ the scalar product) may represent A with equal probability $\eta(A)$. The vector a may be called the macroscopic orientation vector of A. The transmission probability $p(A, B)$ for two realistic polarizers A and B orientated in the a- and b-direction respectively is given by the formula

$$p(A, B) = \eta(A)\eta(B) \int_{O_A} dy \int_{O_B} dx\, p(y^+, x^+) . \qquad (2)$$

In a similar way one may obtain the probabilities $p_{12}(a, b)$ from formula (1). We see at once from (2) that $p(A, A) \neq 1$ which agrees with the fact that the efficiency of realistic polarizers is never 100%. Thus even if we knew a particular spin function we could not predict that the corresponding particle will be transmitted by A. This is the reason why, for a given pair of particles 1 and 2 in an EPR-type experiment a transmission of the particle 1 by A orientated in the a-direction does not imply that the particle 2 has to be absorbed if analyzed by another polarizer A orientated in the a-direction.

(2) According to the realistic model the only information we have about the spin function associated with the particle 2, if we know that the particle 1 has been transmitted, is that it takes the value $+\frac{1}{2}$ on some *unknown* vector y' such that $-y' \in O_A$. Because the values taken by the spin functions on various orientation vectors are correlated in a particular way, the analysis of the ensemble of the particles 2 by means of the polarizer B allow to estimate the probabilities $p_{12}(a, b)$ which should agree with the appropriately smeared quantum mechanical prediction (QMP). We can conclude that the QMP is associated with an ensemble of particles and not with a single particle. Moreover the particles described by identical spin functions can be, a priori, members of completely different ensembles. Thus the spins and the magnetic moments seem to be statistical phenomena (see also refs. [9,10]) and the statement that the particles have their spin "up" in the a-direction only tells something about the property of the beam of particles transmitted by the corresponding polarizer A or of the beam correlated with the analyzed beam in the EPR-type experiment. Therefore each particle having a definite value of the spin function

in all directions, has not a definite spin value in all directions, because it cannot be simultaneously a member of different quantum ensembles. This conclusion holds also for the idealized model.

(3) Arguments presented above show that Pitovsky's statement: "Every electron at each given moment has a definite spin in all directions" is unfortunate and out of the context of the P-model. This statement was used in ref. [8] in order to find the contradictions in the P-model. If we analyze the critical arguments we see that according to the *realistic* P-model, none of the qantities $N_n(x, y)$, $N_n(y, x)$, $N_n(z, x)$, $N(A^+C^-)$, $N(A^+B^-C^-:)$ etc. is measurable, meaningful or related to the relative frequencies used to estimate the probabilites $p_{12}(a, b)$ in a particular coincidence experiment. In some sence we agree with Mermin's conclusion, any random denumerable sequence of spin functions drawn from $F_-(y)$ is not a random sequence of spin functions drawn from $F_-(x)$ if $x \neq y$. The other arguments as to why the Bell inequalities cannot be proven for the realistic spin polarization correlation experiments, may be found in ref. [11].

To conclude: we analyzed, contrary to Bohr's recommendation [5,12], the quantum phenomena in terms of the subquantal description provided by the P-model and we found the confirmation of Bohr's ideas of wholeness and complementarity: a value of a physical observable, here a spin projection, associated with a pure quantum ensemble and in this way with an individual physical system, being its member, is not an attribute of the system revealed by a measuring apparatus; it turns out to be a characteristic of this ensemble created by its interaction with the measuring device. In other words QM is a contextual theory in which the values of the *observables* assigned to a physical system have only a meaning in a context of a particular physical experiment. A similar conclusion can be found with different approaches [13,14].

This aspect of quantum theory doe not seem to be adequately taken into consideration in the attempts to construct a supersymmetric quantum field theory and a quantum theory of gravitation. The quantization of these theories is usually performed with the help of the functional integration technique supplemented by a symbolic calculus: "derivation" or "integration" over the grassmanian variables. Being a formal mathematical procedure this quantization leads to interpretational problems if one wants to relate it to the existing orthodox quantum theory of measurement. In our opinion a new careful epistemological analysis of the various models, which have been constructed to realize the idea of unification of all fundamental interactions, is needed.

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