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A New Description of the Initial States.

M. KUPCZYŃSKI

Institute of Theoretical Physics, Warsaw University - Warsaw

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In the scattering experiments one has to consider the initial ensemble E of two-particle states (1 particle from the beam, 1 particle from the target). If E is pure, each pair of particles before scattering has to be in the same two-particle quantum state. It was realized that in most experiments we deal with initial pairs in different spin states. Thus E is in general not pure and in the theoretical description of the scattering one has to describe this ensemble by a spin density matrix. In spite of the fact that the range of the strong interactions is finite and all the beam and target particles have some small spread in their linear momenta, it was assumed that with respect to other quantum numbers E is pure. This supposition was supported by the independence of the counting rates of the beam intensity and the geometry of the experiment checked in many experiments. However, in our opinion the usual tests for the purity (*) of E were insufficient and a careful experimental and theoretical study of E is needed. Let us imagine that the purity tests will show that E is mixed, how we could find that it is not only an effect of spin? The answer is that one believes that the total cross-sections, inclusive cross-sections, ratios of the different-channel total cross-sections and the multiplicities of the final particles are practically spin independent. Thus the change of the initial mixture in the purity tests should not influence the estimated values of these quantities. On the other hand, if E is mixed, for example, with respect to the relative orbital-momentum quantum number, the measured values of the above-mentioned quantities should depend strongly on the initial mixture.

The idea that E is a mixed ensemble with respect to some additional parameters was expressed in the papers [1,2].

In this paper we want to develop the idea from paper [2] without introducing new parameters for the two-particle states.

Our hypothesis is the following:

1) E is a mixed quantum ensemble with respect to a set of quantum numbers $\alpha = \{S, \Lambda, l, \sigma\}$ where S denotes the total invariant spin, Λ its projection on the direction of the total linear momentum, l the relative orbital momentum and σ the resultant spin of the particles. Thus the initial states have to be represented by the density

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(*) The purity with respect to other quantum numbers than spin.

(1) P. H. EBERHARD: *Should unitarity be tested experimentally?*, CERN 72-1.

(2) M. KUPCZYŃSKI: *Lett. Nuovo Cimento*, **9**, 134 (1974).

matrix $\hat{D}^i$

(1) $$\hat{D}^i = \sum_\alpha |\alpha, \beta, \gamma\rangle \varrho(\alpha) \langle \alpha, \beta, \gamma| ,$$

where the states

(2) $$|\alpha, \beta, \gamma\rangle = |f_\gamma, S, \Lambda, l, \sigma, \beta\rangle$$

and

(3) $$|f_\gamma, S, \Lambda, l, \sigma, \beta\rangle = \int f_\gamma(\boldsymbol{p}', w') \frac{d^3\boldsymbol{p}' dw'}{\sqrt{\boldsymbol{p}'^2 + w'^2}} |\boldsymbol{p}' \Lambda[w', S] l\sigma, \beta\rangle$$

with

(4) $$\int f_{\gamma'}(\boldsymbol{p}', w') f_\gamma(\boldsymbol{p}', w') \frac{d^3\boldsymbol{p}' dw'}{\sqrt{\boldsymbol{p}'^2 + w'^2}} = \delta_{\gamma\gamma'}$$

and the functions $f_\gamma(\boldsymbol{p}', w')$ are very narrow wave packets centred on the $\boldsymbol{p}' = \boldsymbol{p}$ and $w' = w$, where $\boldsymbol{p}$ is the average total linear momentum of the collision and w is the average invariant mass, the label $\gamma = \gamma(\boldsymbol{p}, w)$; the orthogonal vectors $|\boldsymbol{p}' \Lambda[w', S] l\sigma, \beta\rangle$ are standard basic vectors [3] of the type

(5) $$|\boldsymbol{p}' \Lambda[w', S] l\sigma, \beta\rangle = \sum_{\lambda_1 \lambda_2} R L_{\boldsymbol{z}} C(l\sigma; \lambda_1 \lambda_2) \int dR' \, \boldsymbol{D}^s_{\Lambda \lambda_1 - \lambda_2}(R') R' |\boldsymbol{p}\lambda_1, -\boldsymbol{p}\lambda_2, \beta\rangle ,$$

where R and R', $L_{\boldsymbol{z}}$, β, $C(l\sigma; \lambda_1 \lambda_2)$, $|\boldsymbol{p}\lambda_1, -\boldsymbol{p}\lambda_2, \beta\rangle$ denote appropriate rotation Lorentz boosts, internal unitary quantum numbers, numerical coefficients and the individual helicity (in the overall centre of mass) generalized eigenvectors, respectively. In this paper we shall not need the detailed knowledge of the state $\hat{D}^i$. It is enough to say that the states $|\alpha, \beta, \gamma\rangle$ form a convenient orthogonal basis for the description of the initial states and the probabilities $\varrho(\alpha)$ are constrained to fit the probability distribution of the linear momenta and polarizations of the initial particles. It is possible that during the experiment $\varrho(\alpha)$ are time dependent.

2) The formula (1) and the usual arguments (used for the justification of cutting off the partial expansions of the scattering amplitude) that one can neglect strong interactions of the particles in the state with $l > pa$, where p denotes the average relative linear momentum and a the effective range of the strong interactions, lead us to a new representation of the $\hat{S}$-matrix. Namely we make one additional step, *we assume that particles in the initial state with* $l \leqslant pa$ *which are in the range of the strong interaction have always to interact strongly* (*a hadron is not a Swiss cheese*). Of course the forward elastic scattering is possible. In view of these assumptions a block diagonal representation of the $\hat{S}$-matrix seems to be natural:

(6) $$\hat{S} = \hat{I}^{\sim} \otimes \hat{S}^{\sim} ,$$

where $\hat{S} = \hat{I}^{\sim}$ (*identity operator*) *on the subspace of the two-particle Hilbert space* $\mathscr{H}$ *spanned by the vectors* $|\alpha, \beta, \gamma\rangle$ *with* $l > pa$ *and* $\hat{S} = \hat{S}^{\sim}$ (*isometric scattering operator*) *on the subspace of* $\mathscr{H}$ *spanend by the vectors* $|\alpha, \beta, \gamma\rangle$ *with* $l \leqslant pa$.

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[3] J. WERLE: *Relativistic Theory of Reactions* (Amsterdam and Warszawa, 1966), p. 231.

More explicitly

$$(7)\qquad S_{\alpha'\alpha}=\begin{cases}\delta_{\gamma'\gamma}\delta_{A'A}\delta_{s's}\delta_{l'l}\delta_{\sigma'\sigma} & \text{for } l>pa\,,\\ \delta_{\gamma'\gamma}\delta_{A'A}\delta_{s's}S^{\sim}(S,A,\gamma)_{l'\sigma'\beta'l\sigma\beta} & \text{for } l\leqslant pa\,,\end{cases}$$

$$(8)\qquad S_{\varkappa\alpha}=\begin{cases}0 & \text{for } l>pa\,,\\ S^{\sim}_{\varkappa\alpha} & \text{for } l\leqslant pa\,,\end{cases}$$

where $|\varkappa\rangle$ denotes some basis vectors in the N-particle Hilbert space for $N>2$. Here it is good to comment what we understand by the range «a». The range «a» is a Poincaré-invariant quantity defined as follows: for each $\varepsilon>0$ there exists $l_0(\varepsilon)$ such that for each state Φ from $\mathscr{H}$ the probabilities $|\langle\Phi|\hat{S}-\hat{I}|\alpha,\beta,k\rangle|^2<\varepsilon$, if $|\alpha,\beta,\gamma\rangle$ has $l>l_0(\varepsilon)$. Choosing some small ε (depending on the experimental accuracy), we find $l_0(\varepsilon)$ and $a=l_0(\varepsilon)/p$, for cutting off the partial-wave expansions «a» has been taken to be 10^{-13} cm. There is no other way except for the contact interactions between rigid bodies to define the range of the interaction. Only the effective range has a physical meaning. In a similar way the electromagnetic part of the $\hat{S}$-matrix has been eliminated.

3) From 1) and 2) follows that the probability of finding the final particles in the state Ψ is equal to

$$(9)\qquad P_\Psi=\mathrm{Tr}\,(\hat{S}\hat{D}^i\hat{S}^\dagger|\Psi\rangle\langle\Psi|)\,,$$

$$(10)\qquad p_\Psi=\sum_{\substack{\alpha\\ l\leqslant pa}}\varrho(\alpha)|\langle\Psi|\hat{S}^{\sim}|\alpha\rangle|^2+\sum_{\substack{\alpha\\ l>pa}}\varrho(\alpha)|\langle\Psi|\alpha\rangle|^2\,.$$

The total probability p_Ψ is the sum of the incoherent contributions and, *since the interference between the free motion and the forward strong-interaction amplitudes is missing, so one cannot prove the optical theorem* in a complete analogy to the model from [2].

If our hypothesis turns out to be correct, it will require very serious changes in the strong-interaction elementary-particle physics.

On the other hand the mixed character of the initial state could give nice support to all these models of high-energy p-p collisions which use the idea of incoherent summation of the probabilities of all the different processes leading to the same final state. In particular we think about the successful independent cluster production models, discussed for example in papers [4-6] and the references cited therein. The number of clusters emitted by the excited proton in a collision could depend on the value of α characterizing the initial protons in this particular collision.

We treat the hypothesis presented in this paper as some preliminary model hypothesis which should be made more specific if the mixed character of the initial ensemble E of states was found. The tests for the purity of E will be discussed in the subsequent paper.

Let us notice that, if the data indicating the possibility of the violation of the optical theorem [7] were confirmed, then the hypothesis presented in this paper would give a natural explanation of that fact.

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(4) P. Pirila and S. Pokorski: *Phys. Lett.*, **43** B, 502 (1973); *Lett. Nuovo Cimento*, **8**, 141 (1973).
(5) S. Pokorski and L. Van Hove: *Nucl. Phys.*, **60** B, 379 (1973) and preprint TH. 1772-CERN.
(6) A. Białas, K. Fiałkowski and K. Zalewski: *Phys. Lett.*, **45** B, 337 (1973).
(7) M. Kupczyński: *Phys. Lett.*, **47** B, 244 (1973).

Let us also notice that the mixed character of the initial states of the type discussed in this paper may be also appealing to the people believing in the Landau hydrodynamical picture of the high-energy scattering of the elementary particles. This model has been given increasing attention recently; look, for example, (6) and references therein.

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(6) E. SUHONEN, J. EMBEMBERG, K. E. LASSILA and S. SOHLO: *Phys. Rev. Lett.*, **31**, 1567 (1973).