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## NEW TESTS OF COMPLETENESS OF QUANTUM MECHANICS

Marian Kupczynski

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# NEW TESTS OF COMPLETENESS OF QUANTUM MECHANICS *

Marian Kupczynski **
International Centre for Theoretical Physics, Trieste, Italy,
and
Department of Mathematics, Ecole Normale Superieure Takaddoum,
B.P. 5118, Rabat, Morocco.

ABSTRACT

It is observed that in the theory with supplementary parameters TSP each pure quantum ensemble is mixed with respect to these parameters. New statistical purity tests of quantum ensembles are proposed. Additional arguments are given that the violation of the Bell inequalities does not necessarily mean the violation of the Einsteinian separability.

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** Permanent address: Institute of Theoretical Physics, Warsaw University, Hoza 69, Warsaw, Poland.

About 50 years ago Einstein, Podolsky and Rosen[1] (EPR) posed a question about the completeness of quantum mechanics . They have demonstrated that, if one considers the states of two systems 1 and 2 which interacted in the past and are separated in the future, one finds a paradox: a measurement performed on the system 1, claimed to reduce its wave function, implies the immediate reduction of the wave function associated with system 2 in the space-like separated region. Therefore a measurement performed only on the system 1 determines the state of the system 2.

A simple explanation of the EPR-paradox, which seems to be now generally accepted, is a statistical one; the measurement performed on a particular physical system is not equivalent to the reduction of the wave function,which is simply a passage from the description of the whole ensemble to the description of a subensemble satisfying the additionnal conditions.

The statistical interpretation[2] leads in a natural way to the hypothesis of the supplementary parameters which determine the behaviour of a particular physical system. Many mathematical proofs of the inconsistency of the theories with supplementary parameters (TSP) have been given[3]. In spite of these proofs many TSP have been proposed[4]. The main aim of these models was to reproduce the quantum mechanical predictions.

A new epistemological step was done by Bell[5] who analyzed a large class of TSP and showed that one cannot reproduce all the predictions of quantum mechanics for Bohm's version of the EPR experiment. This idea was developed by Clauser, Horne, Shimony and Holt[6] and the realizable experiments have been proposed and performed. The most sensitive ones were those using the pairs of low-energy photons emitted in certain radiative cascades[7,8]. Further investigations have shown that the crucial assumption needed to prove the Bell-type inequalities was the Einsteinian separability[9] and that these inequalities are valid in broader classes of theories; "objective local theories" or realistic local theories"[10]. The most recent very accurate experiments of Aspect et al.[11] have shown that the quantum mechanical predictions are confirmed. The last experiment was a Bohm-Aharonov[12] type experiment with time-varying analyzers. The experiments seemed to indicate that, if a TSP wants to explain the data, it has to violate Einstein's separability[13].

A new solution to the problem has been recently given by Pitovsky[14] (P). He constructed a local deterministic model able to reproduce all the quantum mechanical predictions for the EPR-type experiments. Before giving a new interpretation of the P-model we want to present our principal idea.

Not being the advocates of any particular TSP we want to indicate new tests, which may be useful to verify the completeness of quantum mechanics.

The main feature of the TSP is that the quantum pure ensembles become mixed statistical ensembles of the individual systems characterized by the different values of these new parameters. There is a principal difference between a pure statistical ensemble and a mixed one. The pure ensemble is homogeneous, a mixed one should reveal a fine structure. To see this point clearly we give here a reasoning leading to the operational definition of the pure state[15] and of purity tests[16].

Let O be a stable source of particles and $\gamma$ a measuring device of some physical observables $\gamma X$. A set $S=\{x_i; i=1,..,m\}$, where $x_i$ denote the measured values of $\gamma X$ for m particles produced by a source O, may be interpreted as a sample drawn from some unknown statistical population of the random variable X associated with the observable $\gamma X$. The probability density function f(x) of X and its cumulative distribution function $F(x) = \int_{-\infty}^{x} f(x')\,dx'$ are unknown, but the mathematical statistics give us the means to estimate their main characteristics from the sampling density function or from the empirical distribution function $F(m,x)$, $F(m,x) = n(x_i \leq x)/m$, where $n(x_i \leq x)$ is the number of observations from S smaller or equal to x.

Let $b_1$ be a beam of $m_1$ particles produced by the source O in the time interval $[t_i, t_i+\Delta t]$ and $S_i$ a sample obtained by measuring $\gamma X$ on the beam $b_i$. We may also obtain other families of the beams $b_i(j)$, where $j$ denotes the j-th beam intensity reduction procedure applied to the beam $b_i$. Measuring $\gamma X$ on the beams $b_i(j)$ we obtain the new samples $S_i(j)$. We state that the beams produced by the source O are pure and described by a pure quantum state, if we can not reject the hypothesis $H_o$ : all the samples $S_i$ and $S_i(j)$ for different values of $t_i$ and $\Delta t$ are drawn from the same unknown statistical population of the random variable X.

There are many statistical non parametric compatibility tests which may be used to verify the hypothesis $H_o$. They were extensively reviewed[17] and the examples of their applications were given in a different context[18]. The purity tests may be used to analyze <u>any</u> beam which should be pure according to quantum mechanics and which is suspected to be mixed, if the hypothesis of the supplementary parameters is considered (one can study for example whether in the Fabrikant- or Janossy-type experiments[19] the interference pattern is built up in a regular way).

If the purity of the quantum "pure" ensembles is confirmed, then the statement that quantum mechanics gives a complete description of the individual systems will be proven. The completeness should be understood in the sense that the only predictable and reproducible characteristic of a physical system is : being a member of a given pure ensemble having the properties predicted by quantum theory.

We now come back to the P-model[14] which we discuss in a new way trying, if possible, to use the same notation. Any spin 1/2 particle is described by a spin function $s \in F_o$, $F_o = \{s_o \circ \beta \mid \beta \in O(3)\}$, where $s_o$ is a function on a set of unit vectors $S^{(2)}$ ($s_o(x) = \pm 1/2$ for $x \in S^{(2)}$) and $s_o \circ \beta$ is a usual composition of the function $s_o$ with a transformation $\beta$ from the orthogonal group $O(3)$. We call an ideal polarizer a device Y characterized by an orientation vector $y \in S^{(2)}$, which transmits a particle if $s(y) = 1/2$ and absorbs it otherwise. After the act of transmission a spin function s of a particle is changed into $s'$ with $s' = s \circ \alpha(y)$, where $\alpha(y) \in SO(2)$ is an unknown rotation around the direction y. If one considers the ensemble $F_+(y) = \{s' = s \circ \alpha(y) \mid s(y) = 1/2, \alpha(y) \in SO(2)\}$ one may define the probabilities $p(y^+, x^+)$ of finding the value $s(x) = 1/2$ for $x \in S^{(2)}$ if $s'$ is randomly chosen from the set $F_+(y)$. For two ideal polarizers Y and X, characterized by the vectors y and x respectively, the probability $p(y^+, x^+)$ may be interpreted as the transmission probability $p(Y, X)$ between these two polarizers (probability for the particle which passed by the polarizer Y to be transmitted by the polarizer X).

To describe the EPR-type experiment one may represent each pair of particles 1 and 2 by the corresponding spin functions $s_1$ and $s_2$ respectively with $s_2 = -s_1$ and $s_1$ randomly chosen from the set $F_o$. With any realistic polarizer $Y_R = A$ one may not associate a unique orientation vector y ; any other vector $y' \in O_A$ $\left(O_A = \{ y' \in S^{(2)} \mid |1-y'.y| < \varepsilon_A \}\right.$, where $\varepsilon_A$ is a small real positive number and $y'.y$ a scalar product of two vectors$)$ may represent A with equal probability $\eta(A)$. Thus a transmission probability p(A,B) between two realistic polarizers A and B is given by the formula

$$p(A,B) = \eta(A)\,\eta(B) \int_{O_A} dy \int_{O_B} dx\; p(y^+, x^+).$$

This is the reason why, for a given pair of particles 1 and 2 in a EPR-type experiment a transmission of the particle 1 by A oriented in the "a"-direction does not imply that the particle 2 has to be absorbed if analyzed by another polarizer A. Thus the spins and magnetic moments seem to be statistical phenomena and the statement that the particles have their spins "up" in the a-direction only tells something about the property of the beam of particles transmitted by the corresponding polarizer A or of the beam correlated with the analyzed beam in the EPR-type experiment. Therefore each particle having a definite value of the spin function in all directions, has not a definite spin value in all directions. This is a new answer to some published objections[20] to the first formulation of the P-model.

The pathological features of the model appear only if one wants to ask what is the probability $q(x,y)$ that $s(x) = 1/2$ and $s(y) = 1/2$ for $x \neq y$, if s is randomly chosen from the set $F_o$, but $q(x,y)$ may not be determined experimentally and thus it does not represent any physically interesting quantity. Let us also observe that to describe the random events in any particular experiment[7,8,11] we do not need to abandon the Kolmogorov axioms of the probability theory. However, the measured probabilities in the different experiments may not be determined by conditionalization from a unique probability space. The last assumption was used in all the proofs of the Bell inequalities[21,22].

Concluding : the theoretical and experimental analysis of the EPR paradox and of Bell's inequalities imposed serious restrictions on the models with supplementary parameters and showed that they have to respect in some way Bohr's idea of complementarity.

We hope that the results of the purity tests proposed above will give a new comprehensive answer to the EPR-question concerning the completeness of quantum mechanics.

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