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IS THE OPTICAL THEOREM VIOLATED?

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In this letter we analyse $\bar{p}p$, pp, $\pi^{\pm}p$, $\pi^{\pm}d$, $K^{\pm}p$ and $K^{\pm}d$ data to check the hypothesis of the break of the optical theorem presented in our previous paper.

In this paper we want to find experimental support for the idea expressed in our previous paper [15] indicating the posibility of the break of the optical theorem in the strong interactions without violating the unitarity of the S matrix.

The optical theorem leads to the following relation

$$(\mathrm{d}\sigma/\mathrm{d}t)_{t=0} \geqslant \sigma_{\mathrm{tot}}^2/16\,\pi\hbar^2, \qquad (1)$$

where $(\mathrm{d}\sigma/\mathrm{d}t)_{t=0}$ denotes the elastic differential cross-section for the unpolarized initial beam taken at $t=0$.

It is quite difficult to show the violation of the inequality (1) since the left-hand side of this inequality can be only extrapolated from the data for small angle elastic scattering. All such extrapolations are uncertain and depend on the experimental cut made in the small t. Besides usually the inequality (1) was treated as the main always true constraint on such an extrapolation.

We analyse the data presented in the compilations of data [4, 7, 8, 13].

For pp the elastic total cross-sections are decreasing, for the beam momentum $p = 12$ GeV/c the lowest value is 11.59 mb and for higher energies are not reported in [13]. The total cross-sections, up to $p = 50$ GeV/c, tend to the constant value around 40 mb.

Enström et al. made in their compilation [13] a good job. They gave their best fits to different authors data for $\mathrm{d}\sigma/\mathrm{d}t$ in the different small t-regions for $t \leqslant 0.4$ (GeV/c)^2. They used the exponential formula $A \exp(-B|t|)$. We reproduce on fig. 1 their plot of the values of $A = (\mathrm{d}\sigma/\mathrm{d}t)_{t=0}$ in the fit for $0.05 \leqslant t \leqslant 0.4$ (GeV/c)^2.

As one can see on fig. 1 for $1 \leqslant p \leqslant 2$ GeV/c the values of A vary between 78 and 1018. The values of B (not seen on fig. 1) vary, in the same p-region, between 8 and 17. At the same time the total cross-sections form more or less smooth curve and their values change from 117 to 90 mb.

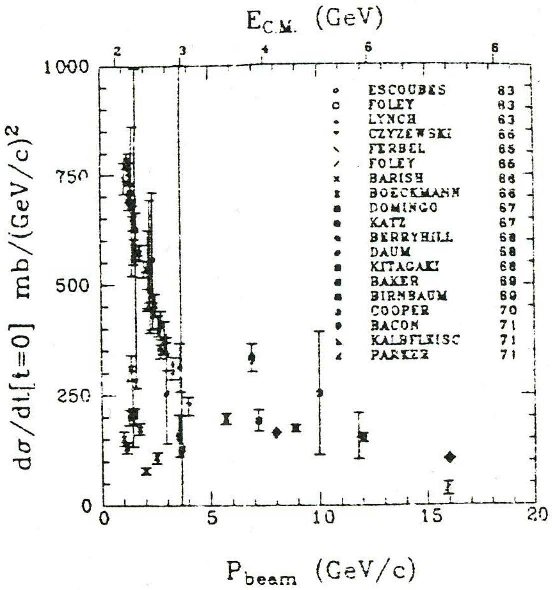

Fig. 1. The values of the $(\mathrm{d}\sigma/\mathrm{d}t)_{t=0}$ from the least - squares fit of the $\bar{p}p$ elastic $\mathrm{d}\sigma/\mathrm{d}t$ to the form $\mathrm{d}\sigma/\mathrm{d}t = (\mathrm{d}\sigma/\mathrm{d}t)_{t=0} \exp(bt)$ over the interval $0.05 \leqslant |t| \leqslant 0.4$ (GeV/c)^2 taken from the compilation Enström et al. [13].

The possible comments are: the data are inconsistent, the cuts in small t are different, the extrapolation procedure can not be trusted. The fits given by some authors differ from those of Enström but they were made with the constraint (1). *The fits without this constraint made in* [13] *for* $0 \leqslant t < 0.4$ (GeV/c)^2 *show bad violation of the optical theorem*. The results are displayed on the table 1. The values of the forward differential cross-sections are from page 85 of

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Table 1

Ref	p beam [GeV/c]	$(d\sigma/dt)_{t=0}$ [mb/(GeV/c)^2]	minimal $\sigma^2_{tot}/16\pi\hbar^2$ [mb/(GeV/c)^2]
[9]	0.450	101.91 ± 11.19	1600
[9]	0.550	9.49 ± 1.07	1350
[9]	0.650	2.67 ± 0.49	1000
[2]	1.125	131.31 ± 12.36	621
[2]	1.137	310.59 ± 29.18	539
[10]	1.430	209.89 ± 76.27	529
[2]	1.500	202.52 ± 17.74	507
[16]	1.510	416.14 ± 7.71	507
[16]	1.650	392.45 ± 6.87	491
[2]	1.750	173.88 ± 13.56	472
[2]	2.000	78.57 ± 8.26	419
[16]	2.450	235.26 ± 6.67	357
[1]	3.550	157.16 ± 47.12	253
[6]	5.700	189.34 ± 10.23	198
[5]	8.000	165.63 ± 0.62	180
[14]	15.910	36.27 ± 15.38	119
[5]	16.000	104.20 ± 1.67	119

ref. [13]. The minimal values of $\sigma^2_{tot}/16\pi\hbar^2$ are calculated using the formula $(\sigma_{tot} - \text{error})^2/16\pi$. Though data which do not show the violation of (1) also exist the numbers in the table 1 have to be taken seriously since they come from many different experiments. Some of them have rich statistics namely: Parker [16] 276 000, Cooper [10] 135 000, Cline [9] 125 000, Boeckmann [6] 15 000 pictures from the hydrogen bubble chambers. The other data [1, 2, 5, 14] are obtained with use of spark chambers and counters.

All these facts indicate that the problem is serious and urgent. A careful analysis of the data and the experiments is needed to confirm or to reject the break of the optical theorem in p̄p interactions shown on the table 1.

The data for the pp scattering are more complicated, the different types of formulae were fitted to the forward peaks. We show on the fig. 2 the values obtained in the best fits to the different authors data only with the use of the formula $d\sigma/dt = (d\sigma/dt)_{t=0} \exp(-b|t|)$ for $0.03 < |t| \leq 0.3$ (GeV/c)^2 made by Bennary et al. [4].

The solid line in fig. 2 is the optical theorem lower limit calculated from σ_{tot}. Bennary makes the following comment: "The two points falling bellow this line (represented by ◇) are from an experiment that starts at a relatively high t value of 0.15 (GeV/c)^2". *However we see 9 points below the optical limit on the fig. 2.* We also see many points above so it is difficult to conclude.

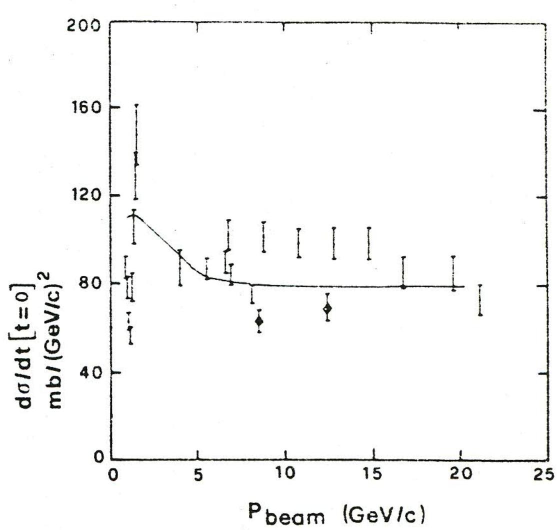

Fig. 2. The values of the $(d\sigma/dt)_{t=0}$ from the least-squares fit of the pp elastic $d\sigma/dt$ to the form $d\sigma/dt = (d\sigma/dt)_{t=0} \exp(bt)$ over the interval $0.03 \leq |t| \leq 0.3$ (GeV/c)^2 taken from the compilation Bennary et al. [4].

For $\pi^\pm$p and $K^\pm$p reactions [7, 8] the total elastic cross-sections are decreasing, the total cross-sections tend to constant values or slightly increase (K^+p) The data for the total elastic cross-sections are not reported in [7, 8] above p = 50 GeV/c for π^-p, above p = 20 GeV/c for π^+p and $K^\pm$p. The analysis of the $d\sigma/dt$ in the observed cases is needed.

A preliminary analysis was done recently by Eberhard [12] who wanted to check his own non unitary model [11]. The need of a careful check of the optical theorem was raised also by Bell [3]. Eberhard fitted the differential elastic cross-sections for π^-p and π^+p between 0.5 and 2.6 GeV/c and between 8 and 26 GeV/c, using the minimum χ^2 method and the Coulomb interference formula. He found the discrepancy of about 5–10% from the optical theorem, namely all forward differential cross-sections are about 5–10% too small. One of the possible explanations Eberhard gave was the break of the optical theorem.

Another remarkable fact is the very poor set of the data on the elastic p̄d, $\pi^\pm$d and $K^\pm$d elastic cross-sections [7, 8].

For $\bar{p}d$ one has two sets of rather inconsistent data on the total elastic cross-sections and only up to $p = 20$ GeV/c. For π^+d and π^-d one has only one experimental point in each case and only for $p \leqslant 4$ GeV/c. For K^-d the data exist only for $p \leqslant 3$ GeV/c and *for* K^+d *there is no data for elastic scattering.*

In our opinion it can be a hint that in those reactions the optical theorem can be violated and should be carefully checked.

Also analysing the ISR data one should consider the possibility of the break of the optical theorem and one should try to use the methods do not depending on its validity.

We do not feel competent to give a comprehensive answer to the question in the title of this paper. However, we hope that our analysis indicates that a possibility of the affirmative answer has to be considered seriously.

A final and more conclusive analysis should be done by the experimentalists who alone know how much the belief in the optical theorem could influence sometimes the analysis of the data (for example the overestimation of the errors).

At the end we want to add a remark how the spin polarization of the initial particles can influence the inequality (1).

One can prove that for the initial state described by an arbitrary spin matrix ρ the following inequality holds

$$(\mathrm{d}\sigma/\mathrm{d}t)^{\rho}_{t=0} = (\sigma^{\rho}_{\mathrm{tot}})^2/16\pi\hbar^2 , \qquad (2)$$

where $(\mathrm{d}\sigma/\mathrm{d}t)^{\rho}$ and $\sigma^{\rho}_{\mathrm{tot}}$ denote appropriate differential and total cross-section respectively. Of course their values can depend on ρ so if one compares $(\mathrm{d}\sigma/\mathrm{d}t)^{\rho_1}$ and $\sigma^{\rho_2}_{\mathrm{tot}}$ one can, in principle, get the violation of the inequality (2). However, σ_{tot} is not expected to depend on the initial spin states what is supported by the experiments. Thus since $\sigma^{\rho_1}_{\mathrm{tot}} \approx \sigma^{\rho_2}_{\mathrm{tot}}$ the violation of the inequality (1) due to neglect of the spin effects can be very small and cannot explain the large descrepancies reported in the table 1.

At the very end we want to elaborate on the inconsistency of the data for $\bar{p}p$ [13] mentioned above. This inconsistency is very serious as seen from the fig. 1. On the table 1 the data violating the inequality (1) are displayed. However, there is a large number of data which do not violate this inequality [13]. Moreover, there are sometimes just a few tens of MeV/c between the beam momentum at which the measured value violates the inequality (1) drammatically and another one where the measured values do not violate it. If this inconsistency were confirmed by the future analysis it could confirm our impurity idea from the paper [15]. In the different experiments we deal with different statistical mixtures with respect to the uncontrolled parameter ξ [15]. For the total cross-sections the averaging over the different measurements could smooth the results but for the differential elastic cross-sections the effect has been, presumably, drammatically observed. With this speculation we end the paper and open the discussion.

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