ON THE ADDITIVITY ASSUMPTIONS

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Some misunderstandings concerning the additivity assumption in the quark model are explained. It is shown that the additivity assumption in the form used for the derivation of the polarization predictions is only some, not very well justified algorithm, for making calculations. Two, more consistent, relativistic formulations of the additivity assumption are discussed. The "additivity without spectators" assumption is formulated. Arguments are given supporting Lipkin's claims that searching for the additivity frame has no deeper physical meaning. It is also shown that all the speculations as to which is the additivity frame are misleading.

I. Introduction

The additivity assumption when used in the discussion of the scattering phenomena[1] (based on the non-relativistic impulse approximation) has led to many (quite) good predictions for the total cross-sections. All these predictions, however, can also be obtained from other theoretical assumptions.[2] Other predictions[3,4] were obtained for the polarization phenomena, but their derivation was inconsistent with the relativistic description of spin.

The aim of this paper is to discuss the additivity assumption - A1 used in the derivation of the polarization predictions of the quark model. A1 has never been stated clearly enough and the usual arguments supporting it concerned the non-relativistic motion of quarks inside hadrons. The non-explicitly stated assumptions lead usually to contradictions and misunderstanding. We claim that A1 can be only understood as some algorithm for making calculations postulated but not derived. So one can repeat Lipkin's question[5] as to which model or which theory are tested by those relations which were derived from A1 for the statistical tensors,[4] reaction amplitudes,[6] generalized statistical tensors,[7,8] and which turned out to be consistent with the experimental data.[9]

In this paper we show that there are two different approaches: the relativistic additivity assumption[10,11] A2 and the "additivity without spectators" assumption[12] A3 which are both consistent with the relativistic description of the scattering. They both indicate that A1 can be used for the practical calculations but the interpretations and the properties of the quark-quark amplitudes are now completely different from those of the usual approach.

II. The Assumption A1

Let us consider the following reaction

$$M_1 + B \to M_2 + B^*, \qquad (1)$$

where B denotes $\frac{1}{2}^{\pm}$ baryon, B* - $\frac{3}{2}^{+}$ baryon isobar,

M_1-and M_2-mesons.

A1 can be formulated as a prescription for making calculations in the following way:

1. Write down the spin and unitary spin part of the hadrons' wave functions corresponding to the appropriate SU(6) representations. The symbols denoting SU(6) spinors interpret as representing some quark states. In this way one obtains the following formulae:

$$M_1^{\alpha_1} = \sum_{i,j} A_{\alpha_1}(i,j)\ \xi_i\ \bar{\xi}_j, \qquad (2)$$

$$B^{\alpha_2} = \sum_{i,j,k} C_{\alpha_2}\ (i,j,k)\ \xi_i \xi_j \xi_k,$$

$$B*^{\alpha_3} = \sum_{i,j,k} d_{\alpha_3}\ (i,j,k)\ \xi_i \xi_j \xi_k,$$

   for $i,j,k = 1,\ldots 6$. In the formulae (2) the following notation is used[6]: $\xi_1 = p_+$, .. $\xi_6 = \lambda-$, p_+ denotes the p - quark spin state with projection $\frac{1}{2}$ on the quantization axis, the coefficients $A_{\alpha_1}(i,j),\ldots d_{\alpha_3}$ (i,j,k) are appropriate Clebsch-Gordan coefficients of the SU(6) group.

2. Write the reaction amplitudes for the reaction (1) as follows:

$$f_{\beta_3\beta_4\beta_1\beta_2} = \langle M_2^{\beta_3} B*^{\beta_4} | T | M_1^{\beta_1} B_1^{\beta_2} \rangle = \qquad (3)$$

$$\sum_{i,\ldots w} A^{\beta_1\beta_2\beta_3\beta_4}\ (i,j,l,m,n,q,r,s,t,w)\ \times$$

$$\langle \xi_i \bar{\xi}_j \xi_l \xi_m \xi_n | T |\ \xi_q \bar{\xi}_r \xi_s \xi_t \xi_w \rangle,$$

where $A^{\beta_1\beta_2\beta_3\beta_4}(i,\ldots w)$ are expressed by the appropriate Clebsch-Gordan coefficients of SU6.

3. Use the additivity assumption which states: only one pair of quark states in the B and M_1 respectively can be changed. Other quarks which states are unchanged are called spectators. However, for the reactions with momentum transfer $|t| > 0$ all the quarks in the initial hadrons should adjust their momenta to this transfer so add some unknown formfactor F(t) taking account of this effect. Thus one obtains the following formula in the short-hand notation of paper[6]:

$$\langle \xi_i \bar{\xi}_j \xi_l \xi_m \xi_n | T | \xi_q \bar{\xi}_r \xi_s \xi_t \xi_w \rangle = \tag{4}$$

$$\sum_{c,k} \left(\langle \xi_i \xi_c | T | \xi_q \xi_k \rangle \delta_{jr} + \langle \bar{\xi}_j \xi_c | T' | \bar{\xi}_r \xi_k \rangle \delta_{ij} \right) \delta_{\vec{b}(c)} \delta_{\vec{b}(k)} .$$

The combination of the formulae (3) and (4) give us the decomposition of the reaction amplitudes into some quark-quark amplitudes.

4. Now give the interpretation for the reaction amplitudes $f_{\alpha_3\alpha_4\alpha_1\alpha_2}$ and for the quark-quark amplitudes. Interpret the amplitudes $f_{\alpha_3\alpha_4\alpha_1\alpha_2}$ as the relativistic (transversity, helicity or other) amplitudes, the quark-quark amplitudes as corresponding (transversity, helicity or other) amplitudes for scattering of free quarks with masses being fractions of the particle masses.

From such explicit formulation one easily sees that

A1 is not non-relativistic assumption as has been thought,[12] since one obtains the expression for the amplitudes in the relativistic spin representations. On the other hand one cannot treat the states ξ_i as the relativistic state vectors of free quarks because in that case one couldn't use the additivity, since for $|t| > 0$ the state vectors of the spectator quarks would in general be orthogonal. So A1 is only the algorithm for making calculations. The author of this paper can be also blamed for introducing some misunderstandings because by the analogy to the paper [3,4] he called in [6] ξ_i the transversity wave functions of the quarks. One could ask why to use such a strange assumption A1. The answer can be as follows.

A1 was proposed[3] by analogy with the non-relativistic additivity assumption and led to good experimental predictions. It is quite usual in elementary particle physics that guessed or heuristically motivated algorithms are tested against experiment. When the agreement with the experimental data for various reactions is achieved one starts to search for some deeper explanation of the assumption or if this is not possible one tries at least to formulate this assumption in a way consistent with some general principles.

The formal explanation of A1, consistent with relativity, is given by A2 or A3 which will be discussed in sections IV and V.

Let us now discuss the problem of the additivity frames.

III. The additivity frames.

A1 turns out to be dependent on the choice of the spin quantization frame.

Let us show it more explicitly. From the formulae (3) and (6), using the prescription of the point 4, we obtain the following expression for the transversity amplitudes:

$$f_{\mu_3\mu_4\mu_1\mu_2} = F(t) \sum_{a_i, \alpha_i} C^{\alpha_1\alpha_2\alpha_3\alpha_4}_{a_1a_2a_3a_4}(\mu_i) \qquad (5)$$

$$\times \quad \langle a_3\alpha_3 a_4\alpha_4 | T | a_1\alpha_1 a_2\alpha_2 \rangle,$$

where $\langle a_3\alpha_3 a_4\alpha_4 | T | a_1\alpha_1 a_2\alpha_2 \rangle$ are the transversity amplitudes for the scattering of the free quarks. If we transform the formula (5) to the spin quantization frame $0'$ obtained from the standard transversity frame by the rotation of the rest frames for hadrons by the angles $\varphi_1, \varphi_2, \varphi_3, \varphi_4$ around the normal to the reaction plane we obtain

$$f'_{\mu_3\mu_4\mu_1\mu_2} = \exp\left[i(\mu_3\varphi_3+\mu_4\varphi_4-\mu_1\varphi_1-\mu_2\varphi_2)\right] f_{\mu_3\mu_4\mu_1\mu_2}, \qquad (6)$$

$$\langle a_3\alpha_3 .. | T | a_1\alpha_1 \rangle' = \exp\left[i(\alpha_3\varphi_3+\alpha_4\varphi_4 - \alpha_1\varphi_1 - \alpha_2\varphi_2)\right] \times$$
$$\langle a_3\alpha_3 .. | T | a_1\alpha_1 .. \rangle, \qquad (7)$$

where from

$$f'_{\mu_3\mu_4\mu_1\mu_2} = F(t) \sum_{a_i, \alpha_i} C^{\alpha_1\alpha_2\alpha_3\alpha_4}_{a_1a_2a_3a_4}(\mu_i)$$

$$\times \exp\left\{i\left[(\mu_3-\alpha_3)(\varphi_3-\varphi_1) + (\mu_4-\alpha_4)(\varphi_4-\varphi_2)\right]\right\} \qquad (8)$$

$$\langle a_3\alpha_3 .. | T | a_1\alpha_1 .. \rangle'.$$

If we had used A1 in $0'$ we would have obtained the formula (5′) which differs from the formula (5) only by primes added to the amplitudes $f_{\mu_3\mu_4\mu_1\mu_2}$ and $\langle a_3\alpha_3 a_4\alpha_4|T|a_1\alpha_1 a_2\alpha_2\rangle$. So A1 is frame dependent and, if it is valid, it is valid in some chosen class of spin quantization frames. The frame in which (3) and (4) are valid was called the additivity frame (the a.f.).

The frame dependence of A1 led to many interpretational difficulties and misunderstandings. Some have claimed that the additivity assumption is a simple dynamical assumption on the scattering operator[5,12] so it cannot lead to the frame dependent results, thus A1 is wrong. Others[13] have been trying to speculate which is the additivity frame. All these claims and speculations seem to be wrong. From the explicit formulation of A1 it can be seen that this is not only the assumption on the scattering operator but also on the value of some unknown spin formfactors which in the additivity frame are proportional to the Kronecker δ's[14]. This last statement will be more clear after we have formulated the assumptions A2 and A3. The frame dependence of such assumption is quite natural and doesn't lead to any contradictions.

IV. The assumption A2

The assumption A2 was formulated and extensively discussed in[10,11] so we give here only short discussion of its principles and results.

A2 states: hadrons in the scattering phenomena can be represented in a formal way in their respective rest frames by the SU6 symmetric wave packets of some free particle states with masses being suitable fractions of the hadron masses. These free particles can be called quarks due to their quantum numbers but they may not coincide with the real free quarks which should, if exist, probably have much bigger masses (6-8 GeV). In the hadrons' c.m. frame for the reaction (1) these wave packets are no longer SU6 symmetric since the necessary Lorentz boosts spoil this symmetry. However, they are well defined relativistic state vectors in the transversity, helicity, or other spin representation possessing some additional degrees of freedom. For example one obtains

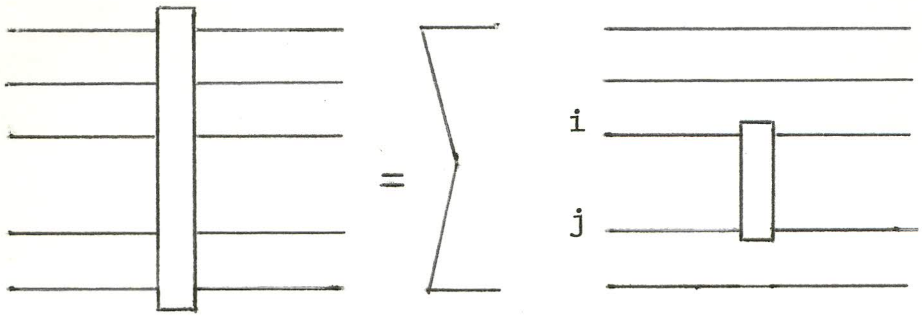
$$M_1^{\beta_1}(\vec{p}) = L(\vec{p}) \int \sum_{\gamma_i, a_i} C^{a_1 a_2}_{\beta_1} (M_1; \gamma_1 \gamma_2) \times \tag{9}$$

$$|\vec{q}_1 \gamma_1 a_1\rangle \otimes |\vec{q}_2 \gamma_2 a_2\rangle \, f_{M_1}(\vec{q}_1, \vec{q}_2)\, \delta_3(\vec{q}_1 + \vec{q}_2)\, d_3\vec{q}_1 d_3\vec{q}_2$$

where $L(\vec{p})$ is an appropriate Lorentz boost, $C^{a_1 a_2}(M_1; \gamma_1 \gamma_2)$ are Clebsch-Gordan coefficients of SU6, $f_{M_1}(\vec{q}_1, \vec{q}_2)$ is rotational invariant weight function[11]. A2 is the following assumption for the scattering operator acting in the vector space L being a tensor product of five free quark state spaces:

$$T = \sum_{\substack{i=1,2,3 \\ j=4,5}} t_{ij} \otimes I_{ij} \tag{10}$$

where t_{ij} is the scattering operator for two free quarks "i" from the baryon and "j" from the meson , I_{ij} is the identity operator acting in the remaining three-quark subspace of the space L . Let us visualize the decomposition (10) with the help of the following diagram:



(11)

$i = 1,2,3$
$j = 4,5$

If the weight functions f_{M_1}, f_{M_2}, f_B and f_{B^*} do not vanish only for $|\vec{q}/w| \ll 1$, then for $|M_H/p| \ll 1$ (where $\vec{q}$ denotes the linear quark momentum in the rest frame of the respective hadron, w - the rest mass of the quark, M_H rest mass of the hadron, p - the c.m. linear momentum of the hadron) one can obtain for the transversity amplitudes a formula very similar to the formula (5):

$$f_{\mu_3\mu_4\mu_1\mu_2} = \exp\left\{\left[\frac{i}{2}\, p \sin\theta \left(\frac{\mu_1+\mu_3}{M_B} + \frac{\mu_2+\mu_4}{M_M}\right)\right] - i\left[\pi\mu_2 + \theta\mu_3 + (\pi+\theta)\,\mu_4\right]\right\} \times \tag{12}$$

$$\sum_{a_i,\alpha_i} C^{\alpha_1\alpha_2\alpha_3\alpha_4}_{a_1a_2a_3a_4}(\mu_i)\, \widetilde{\langle a_3\alpha_3a_4\alpha_4 | T | a_1\alpha_1a_2\alpha_2\rangle},$$

where $\widetilde{\langle a_3\alpha_3a_4\alpha_4 | T | a_1\alpha_1a_2\alpha_2\rangle}$ are some relativistic averaged free quark amplitudes with well defined properties[11] that the b - and c - type relations[4,6] can be valid only very approximately and also for very small momentum transfer. For larger momentum transfer A2 suggests violation of the b and c - type relations and for still larger values of $|t|$ it leads to a serious break down of the usual A1 predictions. If b- and c- type relations are valid for larger $|t|$, they are valid for some unknown dynamical reasons. From the formula (12) it is seen that the exponential factor can be interpreted as a simple spin formfactor and that one could obtain (12) from A1 used in some fixed transversity frame $0'$ specified by the angles φ_i[15]:

$$\varphi_1 = -\varphi_3 = \tfrac{1}{2}\left(\frac{p \sin \theta}{M_M} + \theta\right) \qquad (13)$$

$$\varphi_2 = -\varphi_4 = \tfrac{1}{2}\left(\frac{p \sin \theta}{M_B} + \theta\right). \qquad (14)$$

V. <u>The assumption A3</u>

A3 states: hadrons in the S-matrix description of the scattering phenomena can be represented in a formal way in its rest frames by the SU6 symmetric combination of the free quark rest states. For example we have:

$$M^{\mu_1}(0) = \sum_{i,j} a_{\mu_1}(i,j)\; \xi_i(0)\; \bar{\xi}_j(0), \qquad (15)$$

where

$$\xi_i\left(\frac{\vec{p}_k}{n}\right) = \left|[w\ \tfrac{1}{2}]\ \frac{\vec{p}_k}{n}\ \alpha_i\right\rangle \otimes \left|a_i\right\rangle \qquad (16)$$

α_i and a_i are appropriate spin and unitary spin variables w is the quark mass being 1/3 of the baryon's mass and ½ of the meson's mass.

In the overall c.m. frame state vectors of hadrons lose their SU6 symmetry and can be written

$$M^{\mu_1}(\vec{p}_1) = \sum_{i,j} a_{\mu_1}(i,j)\, \xi_i\left(\frac{\vec{p}_1}{2}\right) \bar{\xi}_j\left(\frac{\vec{p}_1}{2}\right). \tag{17}$$

We will formulate A3 for the transversity amplitudes but it can be formulated in any spin quantization frame. Using the formula (17) one can write the transversity amplitude for the reaction (1) in the following way:

$$f_{\mu_3\mu_4\mu_1\mu_2} = \langle M_2{}^{\mu_3}(\vec{p}_3)\, B^{*\mu_4}(\vec{p}_4)\, |T|\, M_1{}^{\mu_1}(\vec{p}_1)\, B_1{}^{\mu_2}(\vec{p}_2)\rangle =$$

$$\sum_{i,..w} a_{\mu_1\mu_2\mu_3\mu_4}(i,..w) \times \tag{18}$$

$$\langle \xi_i\left(\frac{\vec{p}_3}{2}\right)\bar{\xi}_j\left(\frac{\vec{p}_3}{2}\right)\xi_l\left(\frac{\vec{p}_4}{3}\right)\xi_m\left(\frac{\vec{p}_4}{3}\right)\xi_n\left(\frac{\vec{p}_4}{3}\right)|T|\,\xi_q\left(\frac{\vec{p}_1}{2}\right)\bar{\xi}_r\left(\frac{\vec{p}_1}{2}\right)\xi_s\left(\frac{\vec{p}_2}{3}\right)\xi_t\left(\frac{\vec{p}_2}{3}\right)\xi_w\left(\frac{\vec{p}_2}{3}\right)\rangle$$

Now we assume that the interaction is a two-stage process going through some virtual intermediate states. We write the operator T in the following form:

$$T = T_2 T_1 \tag{19}$$

where T_1 is the operator of the type (10), and the meaning of the operator T_2 is clear from the explicit form of the matrix element:

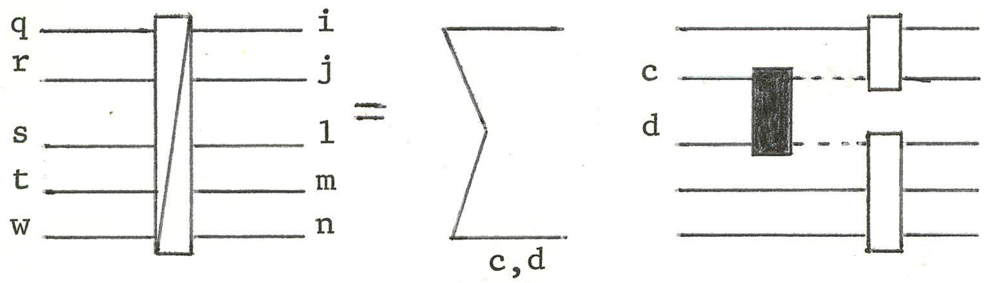
$$\langle \xi_i\left(\frac{\vec{p}_3}{2}\right)..\xi_n\left(\frac{\vec{p}_4}{3}\right)|T|\,\xi_q\left(\frac{\vec{p}_1}{2}\right)..\xi_w\left(\frac{\vec{p}_2}{3}\right)\rangle = \iint d_4q_1 d_4q_2$$

$$\sum_{s,q} \langle \eta_i(q_1)\ \eta_l(q_2)\ |\tau|\,\xi_q\left(\frac{\vec{p}_1}{2}\right)\ \xi_s\left(\frac{\vec{p}_2}{3}\right)\rangle \times \tag{20}$$

$$\langle \xi_i\left(\frac{\vec{p}_3}{2}\right)\bar{\xi}_j\left(\frac{\vec{p}_3}{2}\right)|T_M|\,\eta_i(q_1)\bar{\xi}_r\left(\frac{\vec{p}_1}{2}\right)\rangle \times \langle \xi_l\left(\frac{\vec{p}_4}{3}\right)\xi_m\left(\frac{\vec{p}_4}{3}\right)\xi_n\left(\frac{\vec{p}_4}{3}\right)|T_B|$$

$$\eta_l(q_2)\xi_t\left(\frac{\vec{p}_2}{3}\right)\ \xi_w\left(\frac{\vec{p}_2}{3}\right)\rangle.$$

The decomposition can be represented by the following diagram:



(21)

$c = q,r$
$d = s,t,w$

The diagram (21) represents a two-stage scattering process. This process can correspond in our formalism to the following physical process. At first two quarks from the different hadrons interact with each other (black bubble) next we have an interaction between pairs or triples of quarks belonging to the same hadrons (white bubble). In the first interaction all the charge exchanges and spin flips take place, in the second changes of the four momenta to the final physical values occur.

One obtains the additivity without spectators, if one assumes that for fixed value of s and t there exists such a (transversity) spin quantization frame in which all the unknown spin formfactors are proportional to the Kronecker δ's so:

$$\langle \xi_i \left(\frac{\vec{p}_3}{2}\right) \bar{\xi}_j \left(\frac{\vec{p}_3}{2}\right) | T_M | \eta_i (q_1) \bar{\xi}_r \left(\frac{\vec{p}_1}{2}\right) \rangle = \delta_4 \left(p_3 - q_1 - \frac{p_1}{2}\right) \delta_{jr} T_M(s,t), \qquad (22)$$

$$\langle \xi_l \left(\frac{\vec{p}_4}{3}\right) \xi_m \left(\frac{\vec{p}_4}{3}\right) \xi_n \left(\frac{\vec{p}_4}{3}\right) | T_B | \eta_l (q_2) \xi_t \left(\frac{\vec{p}_2}{3}\right) \xi_w \left(\frac{\vec{p}_2}{3}\right) \rangle =$$

$$\xi_4 (p_4 - q_2 - {}^2/_3\, p_2)\ \delta_{mt} \delta_{nw}\ T_B(s,t). \qquad (23)$$

The spin quantization frame in which the relations (18), (20), (22), (23) hold can be called the additivity frame. Combining the formulae (18), (20), (22), and (23) one obtains the expression analogous to (5) but with different meaning of the quark-quark amplitudes. There is no reason for the formfactors $T_B(s,t)$ and $T_M(s,t)$ to be the same. The b - and c - type relations would be obtained in this formalism only if some additional relations between q-q amplitudes were assumed but there is no justification for them.

One may wonder how a non symmetric diagram (21) can be compatible with the time reversal invariance of the S matrix. We will discuss this problem now.

Let us consider the reaction A + B → C + D. The time reversal invariance of the S - matrix leads to the following equality[16]:

$$\langle C(\vec{p}_3)\ D(\vec{p}_4)\,|T|\,A(\vec{p}_1)B(\vec{p}_2)\rangle = \langle \tilde{T}A(\vec{p}_1)\tilde{T}\ B(\vec{p}_2)\,|T|\,\tilde{T}C(\vec{p}_3)\tilde{T}D(\vec{p}_4)\rangle, \quad (24)$$

where $\tilde{T}$ is the time reversing operator acting on the initial and final states in such a way, that the initial states are transformed into the appropriate final states, and final states into initial states. In other words $\tilde{T}$ reverses the directions of all the momenta and in some way changes the spin states of all the particles. As we see the equations (24) are only the additional constraints on the operator T, i.e. on the operators T_1 and T_2 and do not imply the symmetry of the diagram (21).

VI. General discussion.

Though the two consistent explanations of A1 have been found the problem which model is tested by the predictions of the A1 is not completely solved yet.

A2 and A3 are only some formal approaches to the S matrix description of the scattering processes. Basic in A2 and A3 has been the introduction of the additional degrees of freedom in the description of the initial and final states and the fact that the state vectors of hadrons in the c.m. frame don't belong to the irreducible representation of the SU6 group. They have to transform irreducibly only in the hadrons' rest frames. Their transformation properties under other symmetry groups haven't been considered, only some substructure of the scattering operator T has been introduced. The transformation properties of the operator T have not been considered either. In spite of this many strong predictions concerning the relativistic scattering amplitudes have been obtained. Such way of proceeding is different from the usual methods (mainly non-relativistic) based on the investigation of the transformation properties of the state vectors and the scattering operator with the use of the Wigner-Eckart theorem.

Next we should like to discuss the problem of the connection of A2 and A3 with the quark model. It can be easily seen that such formal approaches may be connected with the quark model but it doesn't have to be. In the quark model, we have been considering, (there are several quark models[17]) the hadrons were treated as strongly bound states of some real physical particles, (which have not been found yet) whose masses are almost completely reduced by the binding forces. Even if the quark were found and the quark model was taken much more seriously it wouldn't be, in our opinion, simple task to prove the possibility of the description A2 and A3 for the relativistic scattering phenomena (of course, if A2 and A3 turn out to be well supported by the experimental data). We see, thus, that the predictions of A2 and A3 cannot test the quark model conclusively, and saying that the testing of them is of vital importance for the quark model [6] is misleading.

Also misleading are all speculations about the direction of spin of the spectator quarks, since in the real scattering such quarks do not exist. In a paper[17] arguments were given that the choice of the Jackson frame as the a.f. restricts the possible shape of the angular distributions for higher spin resonances produced in the meson-nucleon and nucleon-nucleon collisions. Such restrictions occur only if the non-relativistic additivity assumption[18] is used and so they cannot fix the spin quantization frame in which the reaction amplitude for the reaction (1) has the expansion of the type (5) having all the required properties. The a.f. can be fixed by the consistent relativistic assumption A2 or it can be left unknown as in the case of the assumption A3 and each choice is a` priori equally good.

We want to end with some critical remarks concerning A2 and A3. A2 is unambiguous and leads to predictions free of any phenomenological parameters. However, its derivation is valid for restricted values of t and p. The A3 is some kind of ad hoc assumption which gives an a` posteriori explanation of the successes of A1. If the b- and c- type relations are valid they are valid for some dynamical reasons not explained by A2 or A3.

A2 and A3 in some sense generalize the formulation of the additivity of Kokkedee and Van Hove[20]. One can try to build other formal assumptions and one can try to understand the parton models in the similar way.

The careful experimental examination of the predictions of A2 and A3 for the statistical tensors and the generalized statistical tensors (which we would like to recommend as a formalism giving very accurate and detailed tests[7,8] can give answer to the following questions
1) Which one of the additivity assumptions is more reasonable? 2) Are the a, b, and c-type relations valid or not? A positive experimental answer would mean that the approach A2 or A3 is reasonable and gives a convenient parametrization of the reaction amplitudes. Alas, it cannot give us any answer for some deeper theoretical questions such as the validity of the quark model.

There have been proposed some other relativistic assumptions[12,19] but their formulation doesn't seem to us very convincing in view of some arguments given in a section I of this paper.

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